Prediction of tidal currents in the Inner Sound of the Pentland Firth using RTide

Thomas Monahan, Tianning Tang, Stephen Roberts and Thomas A.A. Adcock

Abstract—**The design and operation of tidal stream energy farms will require the accurate prediction of tidal currents from field data. In the present paper we compare the performance of a physics based flow model, the traditional harmonic analysis method, and a newly developed code RTide. RTide is based on the Response Method proposed by Munk and Cartwright in the 1960s and uses machine learning to overcome the key disadvantages of their original approach. We use field data from the Meygen site in the Inner Sound of the Pentland Firth. We find RTide predictions based solely on the field data outperform both the flow model predictions and the currents predicted by harmonic analysis. Interestingly, feeding the physics-based flow model into RTide as an additional input does not outperform the predictions based on the field data alone. However, RTide can still be used as a correction tool to physics-based flow models.**

Index Terms—**Tidal prediction, Response Method, Tidal stream resource**

I. Introduction

THe key advantage of tidal energy over other renewables is that it is, in principle, predictable indefinitely far in advance [1]. Tidal flows are primarily driven by the movements of astronomical bodies which are, for practical purpose, known both in the past or in the future. However, whilst this is true in theory, accurate prediction is challenging in practice. Water levels can in general be predicted accurately although meteorological effects, as well as the disruption from the operation of the barrage itself does provide a challenge [2]. However, the focus of the present paper is on tidal streams where prediction of the currents can be considerably harder. There are two key reasons for this. Firstly, tidal currents are inherently more non-linear than water levels and this means the standard prediction technique of harmonic analysis less suitable for analysis [3]. Secondly, tidal current measurement is expensive and multiple spatial meausrements will be required to build up an accurate prediction of currents across a candidate site. This means that there is a pressue to make predictions from very short time series which again makes predictions more difficult.

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One approach to tidal prediction is to run a physics based flow model, typically solving the shallow water equations [4]–[6] or a layered model [7], [8]. However, these can struggle to accurately model the extreme flows present in a fast tidal race, partly due to the inherent simplifications needed to the physics [9] but also because boundary conditions (i.e. bathymetry and bed drag) are only imperfectly known. As such it is neccesary to analyse field measurements and use these as the basis for predictions. The standard analysis technique for tidal measurements is harmonic analysis. This represents the tidal signal as a number of sinusoidal constituents which can be summed to produce a prediction. A number of implementations exist [10], [11] however, as noted above, this technique is not ideally suited to highly non-linear flows. Some attempts have been made using purely data-driven techniques, which typically extrapolate a time-series [12]–[15]. However, these do not provide any insight into the physics and will have a very limited time horizon.

The approach of the present paper is to use a Response based approach. This builds on the seminal work in the 1960s by Munk and Cartwright [16]. Recent work has used machine learning to recast this approach, developing a code we have called RTide. In principle his approach can tackle far more non-linear problems than traditional harmonic analysis and actually retains more of the inherent physics than the harmonic method. It also naturally lends itself to the inclusion of forcing from non-astronomical factors (i.e. meteorology) although that is not something we explore in the present paper.

We choose to explore this using the Inner Sound of the Pentland Firth as a case study. The Pentland Firth is widely considered to be the most important tidal stream energy site in the world. The site has been studied extensively by numerous authors [17]–[23]. The Inner Sound (colloquially known as the Meygen site) is currently being developed and as such has long-term tidal current measurements and as such is ideal for the present study. In this study we compare predictions of the measured current using various different techniques and analyse the implications for predicting power.

II. Methods

A. Response Method

The response method exploits the fact that the response to the gravitational potential $V(t)$ produced by the Moon and Sun is weakly nonlinear and influenced by past, present, and future values of the tidal potential [16]. To capture this, the potential is expanded in spherical harmonics as

$$V(\theta,\lambda;t) = g\sum_{n=0}^{n}\sum_{m=0}^{n}\left[a_n^m(t)X_n^m(\theta,\lambda) + b_n^m(t)Y_n^m(\theta,\lambda)\right], \tag{1}$$

where g is the gravitational constant, $a_n^m(t)$ and $b_n^m(t)$ denote the complex amplitudes associated with the gravitational potential, and X_n^m and Y_n^m are spherical harmonics at respective order m and degree n.

The objective of the response analysis is to learn the transfer function, defined by learned responsed weights $w_n^m = x_n^m + iy_n^m$, between the tidal potential $V(t)$ and the observations $\zeta(t)$. The linear response prediction is constructed by convolving the learned weights with the expanded complex gravitational potential $c_n^m = a_n^m + ib_n^m$, yielding

$$\hat{\zeta}(t) = \sum_{m,n}\sum_{s}\Big[x_n^m(s)\,a_n^m(t-\tau_s) + y_n^m(s)\,b_n^m(t-\tau_s)\Big]. \tag{2}$$

This formulation, which can be interpreted as a solution to the Laplace Tidal equations after integrating out ocean topography, serves as the foundation for incorporating nonlinear effects.

Nonlinear interactions are introduced by forming sums and products of the linear response terms. In general, the x^{th} order response denoted (R^x) is given by

$$\mathrm{R}^{(x)} = \sum_{i}\cdots\sum_{x}\sum_{s}\cdots\sum_{s'} w(i,\ldots,x,s,\ldots,s')\prod_{j=1}^{x} c(t-\tau_{s_j}) \tag{3}$$

which shows that the number of possible interaction terms grows exponentially with the order of nonlinearity. Traditional approaches often pre-select a subset of these terms—typically by sequentially forming products of the linear components—which assumes that nonlinearity is locally generated and may lead to an underestimation of the variance for higher-order interactions.

For tidal current prediction, the method is extended to a coupled response model that simultaneously addresses the two orthogonal velocity components. The coupled model is written as

$$\hat{\beta}(t) = \begin{bmatrix}\hat{u}(t)\\ \hat{v}(t)\end{bmatrix} = \sum_{m,n}\sum_{s} W_n^m(s)\,c_n^m(t-\tau_s), \tag{4}$$

where the weight matrix $W_n^m(s)$ includes diagonal terms representing the individual responses in the $\hat{U}$ and $\hat{V}$ directions and off-diagonal terms that capture their interactions.

To overcome the challenge of predefining nonlinear interactions, we adopt the machine-learning response method developed by Monahan et al. [24], which employs a three-layer perceptron to automatically learn the complete set of interactions directly from the data using gradient decent. The approach is implemented in the open-source RTide python package.

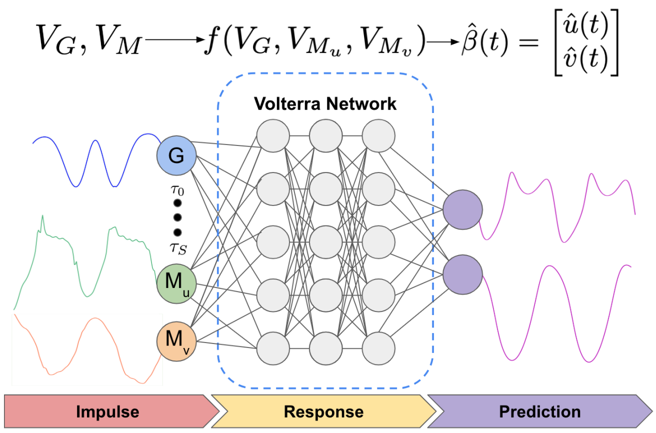

Fig. 1. **Schematic of the coupled Response method for tidal current prediction.** Inputs consist of the lagged gravitational input potential $G(t-\tau_0),\cdots,G(t-\tau_S)$, and optionally the numerical model M_u, M_v.

B. Data

Contiguous data is obtained over 2020 and 2021 from a seabed mounted ADCP located in the Inner Sound. Data is sampled every 10 minutes and was cleaned to remove several erroneous measurements and dropout periods. Concurrent numerical model predictions are provided from a depth averaged MIKE21 model which is operationally used at the Meygen site. At present, this is the standard approach used operationally by developers at the site to characterise the flow. Details of the shallow water model are not available but, given the importance of the data to operational use of the site, we assume that the modelling was carried out to a high standard and based on the best available data and best practice.

C. Models

A description of the models employed for the subsequent case-studies is as follows:

- **RTide:** Standard response model which only takes gravitational forcing as input.
- **RTideM:** Response model which takes MIKE21 model as additional input.
- **Mike:** Raw MIKE21 flow model output.
- **MikeC:** MIKE21 flow model with the empirical correction factor used operationally.
- **UTide:** Standard iteratively-reweighted least-squares harmonic analysis with constituents determined by the record length using a Rayleigh criterion of 1 [10].

III. Case Study

We here look to evaluate our approach against empirical harmonic analysis and the operational MIKE21 flow model used at the Meygen site.

A. Experimental Setup

The dataset is partitioned into train and test sets such that RTide and UTide models are trained on 1.5 years of data from 2020 to mid 2021. Models are then evaluated on unseen data from the second half of 2021.

B. Results

Figure 2 shows 48 hours of predictions on the test-set. Observations have been low-pass filtered with a cut-off frequency of 12 cycles per hour – this low-passed data was also utilized for training. An example of models trained and tested on unfiltered data is provided in the Appendix.

The dominant current at the site is in the east/west (U) direction. This current is strongly semi-diurnal and relatively undistorted. For the observations shown, the UTide can be seen to overestimate peak flow values, with the RTide predictions sitting closer to the observed values. Interestingly, while the MikeC model seems to mimic the distorted profile, these variations can be overestimated, and in some cases be the opposite of the observed modulation. The UTide residuals exhibit a biomodel distribution around zero, and the MikeC and RTide distributions are normally distributed, albeit with the RTide residuals exhibiting lighter tails. This is nicely reflected by Table I, which shows a reduction in RMS error by RTide of 16% and 5% over the UTide and MikeC models respectively. Table I also compares the performance of the uncorrected MIKE (Mike) and RTide MIKE (RMike) models. Interestingly, inclusion of the flow model as an additional input into RTide actually worsens model performance when trained on 1.5 years of data. When using less data, e.g. shorter than 6 months, we do find an improvement in RMike over RTide (results not shown).

The less dominant North/South (V) direction is more revealing as subtle changes in direction are more relevant. The observed time-series shows a clear overestimation of the V component by MikeC model. This yields a heavily skewed residual and a correspondingly high RMS error of 0.234 m/s. In contrast, the UTide predictions tend to severely underestimate the first peak of the quarter-diurnal signal leading to a broad and skewed residual distribution. While this is less egregious than the MikeC predictions, it yields an RMS error of 0.183 m/s. RTide does well to capture the quater-diurnal structure of the signal, leading to a reduced RMS of 0.152 m/s, but misses out on some of the higher frequency peaks. Again, we find that inclusion of the numerical model into the response method actually makes performance marginally worse. The reasons for this degradation in perfromance are unclear.

TABLE I
RMS ERRORS FOR U AND V COMPONENTS, AND TOTAL ERROR MAGNITUDE E_{TOTAL} OVER 180 DAY FORECAST ON TEST-DATA.

Model	u (RMS error)	v (RMS error)	E_{total}
Mike	0.325	0.236	0.402
MikeC	0.305	0.234	0.384
UTide	0.343	0.183	0.389
RMike	0.292	0.154	0.330
RTide	**0.289**	**0.152**	**0.326**

IV. Power Prediction

While flow predictions are interesting for site characterization, the relevant quantity for operational usage is the associated power produced by the flow [12]. We here look to quantify the implications of inaccurate flow predictions on forecasted power production. To do this, we adopt a simplified power curve:

$$P = \begin{cases} 0 & 0 \leq V_t < V_{cut-in} \\ 0.5 C_p \rho A V_t^3 & V_{cut-in} \leq V_t < V_{rated} \\ P_{rated} & V_t \geq V_{rated} \end{cases} \tag{5}$$

where C_p is the power capture coefficient $(0.4 - 0.5)$, ρ is the water density, A is the swept area of the blades (81π), V_t is the tidal current speed, V_{cut-in} is the minimum current speed for turbine operation $(1\,\mathrm{m/s})$ and V_{rated} is the rated operational speed $(3\,\mathrm{m/s})$ [25]. In this idealised analysis we assume that there is only a small impact on the flow from the turbines (see for instance [26]. We transform model observations and predictions to the equivalent normalized power by applying Equation 5 along the dominant channel flow direction (as determined by derived harmonic analysis ellipse parameters).

Figure 3 shows kernel density estimates of the distribution of power prediction errors and Table II shows the total power error over the 180 day test-set. It can be seen in Figure 3 that RTide, UTide, and MikeC models are separated by less than 1 percent across average errors. However, it can be seen that RTide has a much higher estimated density at low power errors. The cause for the departure between average and total power is a consequence of the heavier tails of the UTide and MikeC models. The 95^{th} percentile errors are 15.8%, 20.2%, and 20.9% for RTide, UTide, and MikeC respectively. Since tidal energy production is a cubic function of velocity, even small under- or overestimations in current speed can lead to large power deviations, amplifying the impact of extreme errors. This highlights the importance of not only minimizing mean errors but also understanding the structure of error distributions when evaluating long-term energy yield predictions.

TABLE II
TOTAL NORMALIZED POWER PERCENTAGE ERROR OVER 180 DAY FORECAST ON TEST-DATA.

Model	P_{Error}
Mike	33.1%
MikeC	22.4%
UTide	24.8%
RMike	15.2%
RTide	**12.3%**

V. Discussion

The preceding case-studies demonstrate the advantages of our machine learning response method for the prediction of tidal currents. A clear result from this analysis is that response approaches outperform harmonic analysis across the board. A separate paper (under review) has shown that these improvements actually increase as the amount of data decreases.

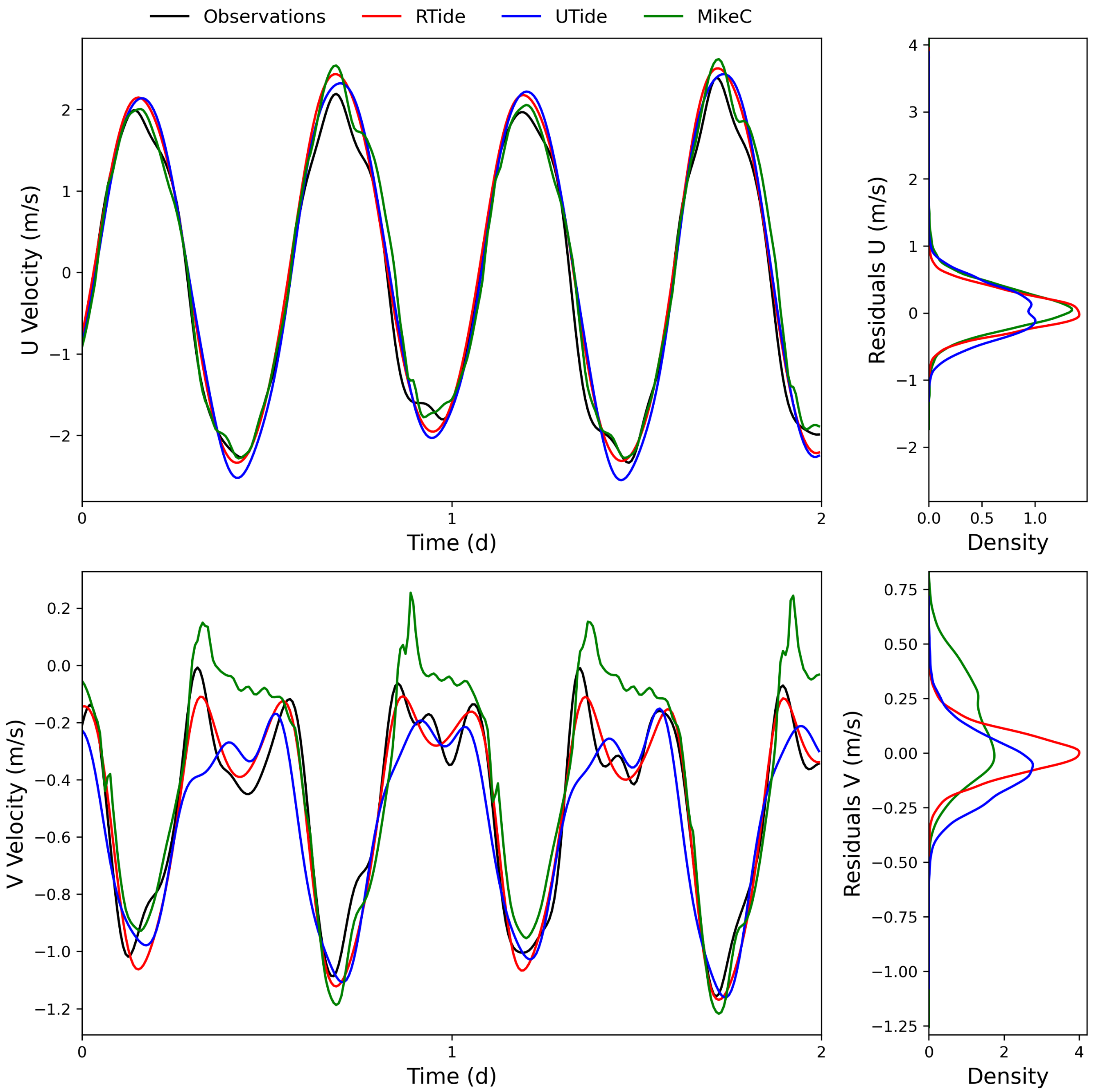

Fig. 2. Comparisons of model predictions on unseen data. V velocity is that in the east/west direction with U velocity in the north south direction. Days selected are representative of the data. Kernel density estimates of the residuals are computed for the entirety of the 180 day test-set. Observations are low-passed filtered with a cut-off frequency of 12 cycles per hour.

Indeed the response approach can make reasonable predictions even when trained on timeseries of less than a month which is particularly valuable for tidal energy applications where it is expensive to deploy ADCPs and there is a trade-off between long measurements at a few locations against shorter measurements at many locations across a site.

We advocate for the replacement of conventional harmonic analysis predictions with response based predictions when accuracy is paramount. A valid criticism of our approach is the lack of interpretability which is naturally afforded by the harmonic framework (for intance it is easy to understand the magnitude of an “M_2 current” or the phase lag between the water level slope and the current for a given constituent. A simple solution is to estimate the harmonic constituents directly from a response method prediction which has been shown to yield superior constituent estimates [24].

Due to the complexity of the flow in tidal energy sites, even well calibrated numerical models exhibit severe errors. Even if we assume model performance to be comparable, a major advantage of the response approach is the computational efficiency. The models trained in this work took approximately 3 minutes to train each with predictions being generated near instantly thereafter. Models were run without modification, and thus required little expertise to setup. In con-

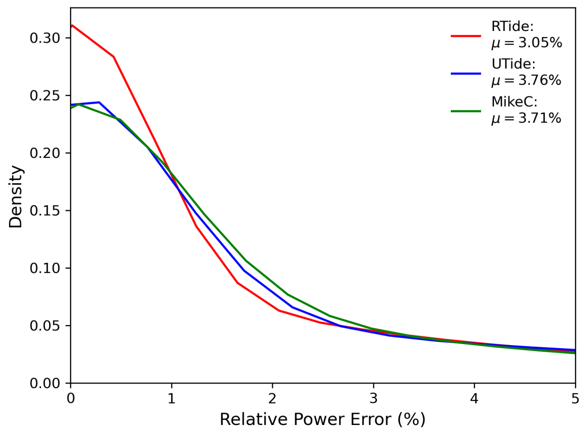

Fig. 3. **Kernel density estimates of normalized power residuals over 180 day test-set.** Mean percentage errors for each model are reported.

trast, numerical models require laborious and costly development as well as access to high-performance computing resources. In the case where no or limited observational data is available, numerical modeling becomes the de facto option. RTide can still be useful in this context as its physical basis can be exploited to learn and emulate the numerical model. In so doing, accurate predictions can be generated at any point in the future, without requiring extensive time-stepping.

In the present paper we have not considered meteorological forcing. However, we note that including forcing from winds, pressure and potentially radiation stresses from waves is a natural extension of a response approach [27]. The same cannot be said for harmonic analysis and whilst such forcing can, of course, be included in flow models, this would require substantial extra computation. A further 'forcing' that could be considered is whether turbines are operational. Tidal turbines must apply a substantial thrust to the flow in order to generate power and how turbines are used will alter the flow. In principle, this is something that could be included in RTide although how well this would work remains an open question. It would also not be possible to validate this in the field until a large tidal turbine farm is deployed.

VI. Conclusion

In this paper we have shown how the open source RTide code can be applied to the real problem of predicting tidal currents at a candidate site for tidal energy extraction. Without modifications to the standard formulation, RTide outperforms alternative models and has the potential to be a valuable tool as the tidal stream industry develops.

Appendix

Figure 4 presents a comparison between the raw (i.e. unfiltered) data and the predictions. The overall conclusions from the data are similar to those presented in the body of the paper.

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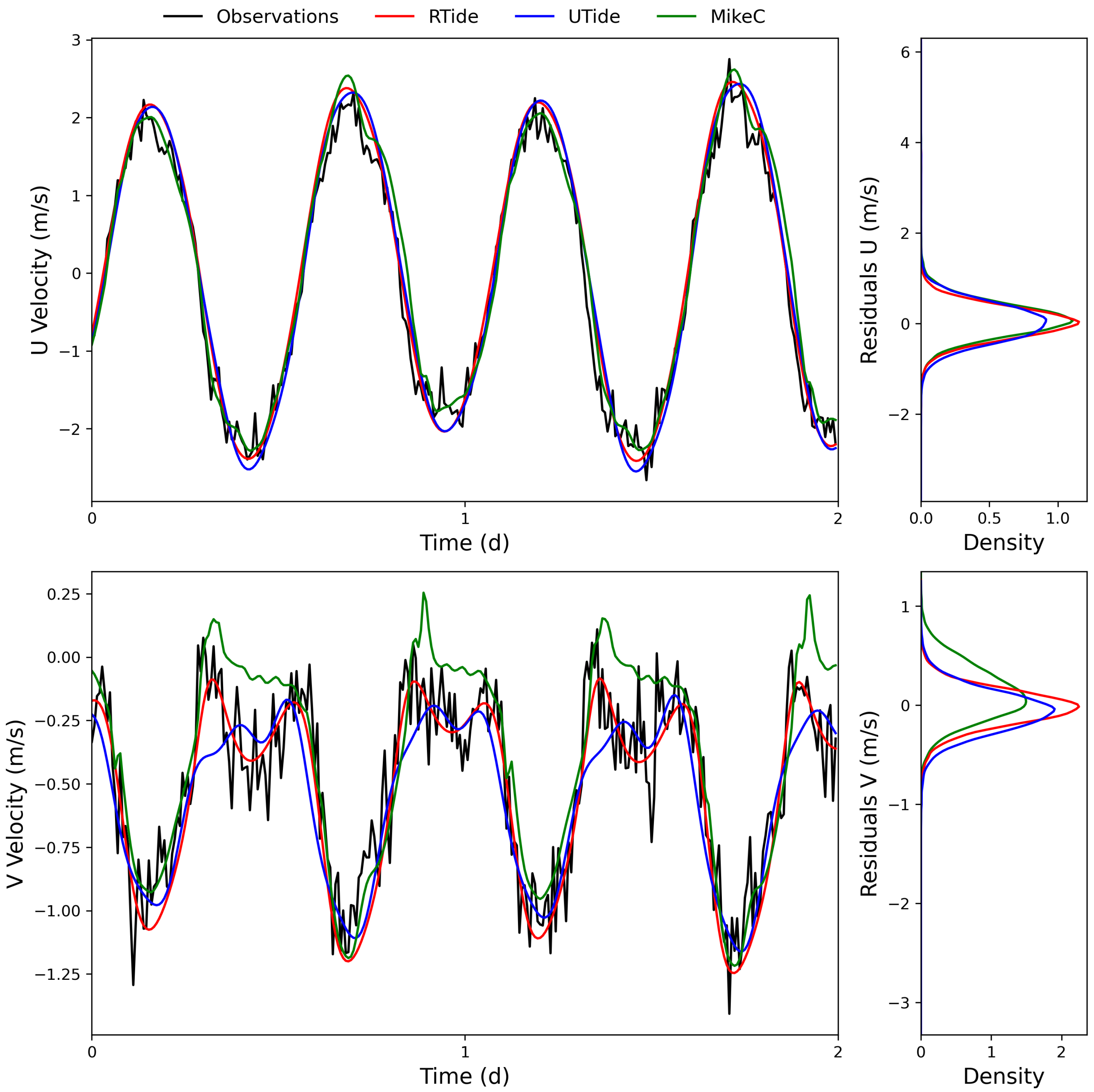

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