Intermediate philosophy of physics: Philosophy of special relativity

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1 Spacetime structure

1.1 Pre-relativistic spacetimes

References:

- J. Earman (1989), chapter 2. (Thorough, but beware of the differential geometry; you can get a pretty good account of what's going on by reading the comprehensible bits and skipping maths that you don't understand.)
- R. Geroch (1978). Part A: The space-time viewpoint. (Less comprehensive and more long-winded, but more accessible than Earman.)

1. Every man and his dog is familiar with the point that special relativity changes our notions of space and time, and that one can try to ask questions, like 'do the London–Oxford and the Oxford–London buses leave at the same time', that are 'meaningless' (suffer from presupposition failure) in the context of special relativity. Further, that the reason this happens is that special relativity attributes less structure to space-time than do pre-relativistic theories.

2. Less familiar is the point that pre-relativistic theories disagree radically *among themselves* about what structure spacetime has. It is helpful to precede our study of spacetime structure in SR with clarity concerning the range of pre-relativistic options. One can identify 'Machian', 'Leibnizian', 'Maxwellian', 'Galilean', 'Newtonian' and 'Aristotelian' spacetimes, each equipped with strictly more structure than the previous.

   Earman's chapter goes through this in detail. Here's a summary; make sure you're able to answer the 'quiz' at the end of Earman's chapter.

Spacetime	New structure	New invariant notions (e.g.)
Machian spacetime	Simultaneity; Spatial metric on each simultaneity slice	Simultaneity; Instantaneous distance
Leibnizean spacetime	Temporal metric	Relative velocity; Relative rotation
Maxwellian spacetime	Standard of rotation	Absolute rotation
Galilean spacetime	Standard of 'straightness' of worldlines	Absolute acceleration
Newtonian spacetime	Standard of absolute rest	Absolute velocity
Aristotelian spacetime	Privileged spatial location	Distance from centre of universe

We can't see spacetime structure directly. It is worth asking: What, then, is the right methodology for working out which spacetime structure we ought to believe in?

The usual answer to this question is: one should believe in just the amount of spacetime structure that is needed in order invariantly to make sense of the distinctions that are required by one's best physics. (For instance, insofar as one believes Newtonian physics, one should believe in *Galilean* — *not* Newtonian — spacetime. This is because the usual equations of Newtonian mechanics are valid only in *inertial* frames, so we do require the 'affine structure' that distinguishes inertial from non-inertial worldlines; Newtonian mechanics does *not*, however, make any use of the notion of absolute velocity.)

1.2 Minkowski spacetime

Reference:

- T. Maudlin (2006). (Elementary — accessible to readers who have not studied SR as part of a physics course.)

1. Special relativity adds another line to the above table. The difference is that we are no longer merely adding to, or subtracting from, the structure of a previous spacetime. We are throwing out (almost) all pre-relativistic structures, including the spatial metrics that are present in all the above spacetimes and the temporal metric that is present in all but Machian spacetime, and replacing them with the *single* piece of structure required by SR — the Minkowski metric.

Spacetime	Structure	Invariant notion
Minkowski spacetime	Minkowski metric	Spatiotemporal interval

2. The Minkowski spacetime interval between two events a, b is given (in Lorentz coordinates) by the expression

$$d(a,b) = \sqrt{(\Delta t(a,b)^2 - \Delta x(a,b)^2 - \Delta y(a,b)^2 - \Delta z(a,b)^2)}, \qquad (1)$$

where $\Delta t(a,b) := t(a) - t(b)$, and *mutatis mutandis* for $\Delta x, \Delta y, \Delta z$.

3. Minkowski diagrams

   - A Minkowski diagram usually suppresses two of the spatial dimensions. The vertical axis is the 't'-axis, the horizontal axis the 'x'-axis, for some Lorentz coordinate system.
   - For most physical phenomena (e.g. the worldline of a particular particle), exactly how that phenomenon looks on a Minkowski diagram depends on which Lorentz chart we singled out to draw the axes. But some structures are *invariant*, i.e. look the same regardless of which Lorentz chart the diagram is drawn with respect to. These invariant structures include the loci of points that are a given Minkowski distance d from a given point p of the spacetime.

4. Hyperboloids

   - In a *Euclidean* space, the set of points at a fixed distance from a given point forms a sphere. ($(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$, for some constants $r, x_0, y_0, z_0 \in \mathbb{R}$.)
   - The set of points a fixed *Minkowski* distance from a given point forms an *hyperboloid*:

   $$\Delta t(a,b)^2 - \Delta x(a,b)^2 - \Delta y(a,b)^2 - \Delta z(a,b)^2 = b, \qquad (2)$$

   for some constant $b \in \mathbb{R}$.
   - What this set of points looks like on a Minkowski diagram depends on whether the constant b is positive, negative or zero.

5. 'Imaginary' Minkowski distance ($b < 0$)

   - The set of points at a given *negative* spatiotemporal squared-distance from a given point p forms an 'hyperboloid of two sheets' (see figure 1).
   - We say that such points are *timelike separated* from the selected point p.
   - The worldlines of ordinary[1] massive particles are everywhere timelike (i.e. each point on a given worldline is timelike separated from all other points on the same worldline).

[1] The caveat 'ordinary' is a nod to the consistency of the existence of 'tachyons' with special relativity.

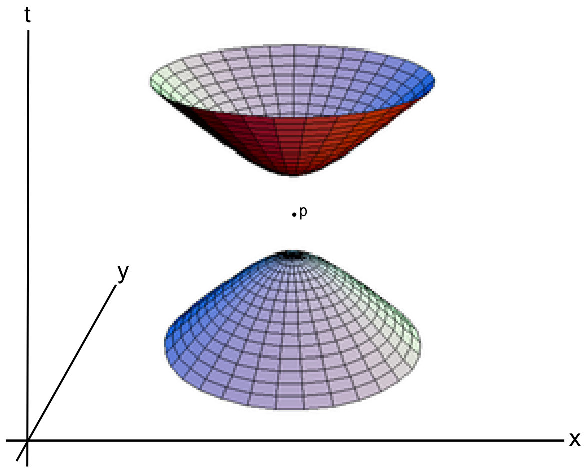

Figure 1: Loci of points a constant negative spatiotemporal distance-squared from a fixed point p.

6. 'Real' Minkowski distance ($b > 0$)

   - The set of points at a given *positive* spatiotemporal squared-distance from a given point forms an 'hyperboloid of one sheet' (see figure 2).

   - These points are said to be *spacelike separated* from p.

7. Zero distance ('b=0')

   - The set of points at *zero* spatiotemporal distance from a given point forms a double cone (a degenerate hyperboloid; see figure 3).
   - The points on the cone are said to be *lightlike separated from* p. They can just be connected to p by means of a light ray, but cannot be connected by the wordline of any massive particle.

2 Standard vs generally covariant formulations of a physical theory

2.1 Introduction

1. We are accustomed (from school and high school physics) to the idea that the equations of one's physical theory are true only in a privileged class of coordinate systems (usually, the 'inertial' systems).

   It is, however, also possible to formulate a physical theory in such a way that its equations are true in an *arbitrary* coordinate system.

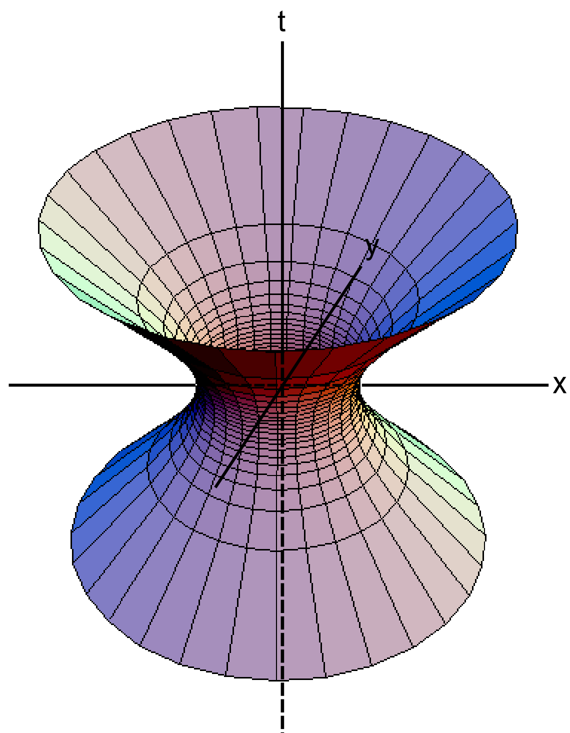

Figure 2: Loci of points a constant positive spatiotemporal squared-distance from a fixed point p.

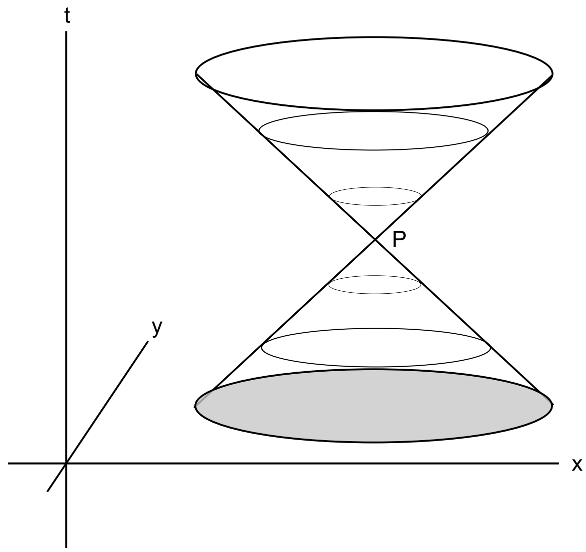

Figure 3: The locus of points zero spatiotemporal squared-distance from a fixed point p.

It will greatly illuminate our understanding of several conceptual issues in the foundations of special relativity (including the ongoing disputes over the conventionality of simultaneity, and over the independent reality of Minkowski geometry) if we understand how this can be done.

For example:

2. Earlier, we encountered the following quote:

> [Newton's first law] reads in detailed formulation necessarily as follows: Matter points that are sufficiently separated from each other move uniformly in a straight line — provided that the motion is related to a suitably moving coordinate system and the time is suitably defined. Who does not feel the painfulness of such a formulation? (Einstein, 1920)

3. And we tried to talk ourselves out of pain by observing that the *existence* of an inertial frame is nontrivial. But there are two reasons one might consider a law like Newton's First painful:

   - Worries about circularity (and hence lack of empirical content);
   - A sense of 'massive cosmic conspiracy'.

   Our observation *at best* removes the first of these pain-sources. We still have a sense of mysterious conspiracy; we lack (that is) any explanation of *why* some coordinate systems are 'better' than others.

4. From this point of view, the point of formulating a theory in a generally-covariant manner is that
   (a) in a generally covariant formulation, by definition, no coordinate system is better than any other, so the above puzzle does not arise;
   (b) it is in understanding the relationship between the generally covariant formulation and the 'standard formulation' of a given theory that one comes to understand why some coordinate systems are 'special' (i.e. we solve the puzzle) in the context of the latter;
   (c) roughly, the explanation is that (as in the above analogy) there is more real structure to the world than is explicitly represented in the standard formulation. The generally covariant formulation explicitly represents all structure required to render the theory non-mysterious (this is how it manages to be generally covariant — coordinates now really are just arbitrary labels).

2.2 Spacetime structure and three styles of theory-formulation

Reference:

- J. Norton, 'Philosophy of space and time', in Merrillee H. Salmon (ed.), *Introduction to the philosophy of science*, Hackett (1999). Sections 5.4–5.7. (Available for download from `http://www.pitt.edu/ jdnorton/homepage/cv.html`.)

As a simple example to illustrate the issues, consider first a very sparse physical theory: we are simply modelling the temporal continuum (which we will call T).

1. The standard formulation[2]
   (a) We use a coordinate system: a mapping $t : T \to \mathbb{R}$ from our temporal continuum T to the real numbers.
   (b) The point of this coordinate system is to encode certain physical facts about the temporal continuum T. For example:
      i. There is a physical fact about the temporal distance $\Delta T(a, b)$ between instants a and b.
      ii. This fact is represented in our coordinate system via the coordinate-dependent expression $\Delta T(a, b) = |t(b) - t(a)|$.
      iii. There is a physical fact (let us suppose) about which of any two distinct instants a, b is *later than* the other.
      iv. This fact is represented in our coordinate system via which of $t(a), t(b)$ is the greater number.

[2]Norton breaks this section of the discussion into two parts, introducing a 'one coordinate system formulation' before moving on to the 'standard formulation'. What appears in these notes is a significantly condensed version of Norton's discussion.

(c) But if our original coordinate system $t : T \to \mathbb{R}$ is a 'good coordinate system' in the sense that the facts about temporal distances and later-than relations can be recovered from the coordinates in this way, then so also is any other coordinate system t' that is related to t by a 'constant shift', i.e. any t' such that for some $k \in \mathbb{R}$, we have

$$\forall a \in T, t'(a) = t(a) + k. \tag{3}$$

(d) So the 'standard formulation' of our theory has two components: (i) formulae for recovering physical facts from coordinate-dependent expressions in the 'good' coordinate systems, and (ii) the formulae that relate one 'good' coordinate system to another.

2. The generally covariant formulation

   (a) So far, we have been *implicitly* representing the facts about temporal differences, by stipulating (i) how temporal distance is to be recovered from coordinates and (ii) which coordinate systems are allowed.

   (b) In a generally covariant formulation, we represent the physical facts *explicitly*, by means of additional fields. This gives us the freedom to use *any* coordinate system.

   (c) It is clear that, if we are allowing arbitrary coordinate systems, we can no longer rely on the coordinate difference $t(a) - t(b)$ to represent the temporal distance between the events a and b.

   (d) What we can do instead: Introduce a 'scale factor', $w : T \to \mathbb{R}$.

   - Simple example: a linearly stretched coordinate system
     - Suppose that $t : T \to \mathbb{R}$ is a standard coordinate system.
     - Let $t' : T \to \mathbb{R}$ be an alternative coordinate system, related to t by
       $$\forall a \in T, t'(a) = 2t(a). \tag{4}$$
     - Then, if a, b are events such that $t(a) = 1000$ and $t(b) = 1001$, we have $t'(a) = 2000, t'(b) = 2002$; so $\Delta t(a, b) \equiv t(b) - t(a) = 1$, and $\Delta t'(a, b) \equiv t'(b) - t'(a) = 2$.
     - Key aspect of this: $\Delta t(a, b) = \frac{1}{2}\Delta t'(a, b)$. So, we need to multiply the coordinate difference $\Delta t'$ by the scale factor $\frac{1}{2}$, in order to recover temporal distances from our 'stretched' coordinate system.
     - For this coordinate system t', the scale factor $w' = \frac{1}{2}$.
   - General case: an *arbitrarily* stretched and squeezed coordinate system
     - Let $t'' : T \to \mathbb{R}$ be an arbitrary coordinate system for the temporal continuum T.

- The 'scale factor' w'' for the coordinate system t'' is given by

$$w'' = \frac{dt}{dt''}w \tag{5}$$

(at every point).
- We then recover temporal distances from a combination of the coordinate system and scale factor as follows:

$$\Delta T(a, b) = \int_a^b dt'' w''. \tag{6}$$

- It is easy to see that this expression for ΔT will give the same answer as our original expression $t(b) - t(a)$ in terms of standard coordinates.

3. How we can now answer the puzzle previously raised

   (a) We started with a standard formulation of our theory of linear time. In this formulation, some coordinate systems were preferred to others. Something real had to be conceived as the cause for this preference of some coordinate systems over others. That something is the *temporal metric*. In the generally covariant formulation of the theory, the temporal metric is represented explicitly, by means of the new field w. It is because the standard formulation does not *explicitly* mention w that it can truly describe the coordinate-independent facts only relative to special coordinate systems (viz., coordinate systems in which w happens to take the value 1 everywhere).

4. Aside (advanced): coordinate-*generality* vs coordinate-*independence*

   (a) Our above ('generally covariant') formulation of the theory is coordinate-*general*: it allows us to construct a representation of the physical facts using *any* coordinate system.

   (b) This is distinct from a coordinate-*independent* formulation. A coordinate-independent formulation would allow us to represent the physical facts using *no* coordinate system.

   (c) It is possible to formulate theories in a coordinate-independent way. (Indeed, once the relevant mathematics is mastered, the coordinate-independent formulation is a trivial rewriting of the coordinate-general version.)

   (d) In a coordinate-independent formulation, we would describe the *intrinsic nature* of the background structures and dynamical fields and the relations they bear to one another, without having to say 'and this is how those facts are represented in such-and-such a coordinate system'. This formulation has the advantage of enabling us to *see why* the coordinate components of vectors, one-forms etc each transform in their characteristic ways under changes of coordinate system.

- Example 1 (vectors in a 3D space): By understanding that a given triple of numbers *represents an arrow in spacetime*, as opposed to (say) a set of three scalar fields, we are able to *predict* how it will transform under rotations of our coordinate system.

(e) This coordinate-independent approach greatly illuminates the position of realism about spacetime structure, and the reasons why its advocates find it so compelling. Unfortunately, a full understanding of it this requires the mathematical machinery of differential geometry, which we don't expect you to have studied.

- The above 'scale factor' is, in the language of differential geometry, a one-form.

(f) For those interested (and willing to battle with the maths!), the *locus classicus* of the spacetime-structure-realist approach is (Friedman, 1983). Another classic discussion in the same spirit is (Earman, 1989); see especially chapters 2 and 3.

2.3 Generalising the lessons: other structures

In our first example, we considered a one-dimensional 'space' (viz. time), and we supposed there were facts about which of any two intervals was the larger, and which of any two points (instants) was the later.

We consider now a variety of other spaces (including spacetimes) that have played a role in physics. The aim in each case will be (i) to review the standard formulation, with which you will already be familiar, and then (ii) to see how we can express the same space-theory in a generally covariant way. In each case, we will find that we can move to a generally covariant formulation by introducing extra fields — fields representing the space(-time) structure that the standard formulation treats as 'background'.

1. Euclidean 3-space

   (a) In a Euclidean 3-space S, there is a fact, for any pair of points, about how far apart those two points are.

   (b) The standard formulation

   - Represent space S via a coordinate system $(x, y, z) : S \to \mathbb{R}^3$.
   - Recover distance facts as follows: For any two points a, b of the 3-space, the distance $d(a, b)$ between a and b is given by

$$d(a,b) = \left((x(a) - x(b))^2 + (y(a) - y(b))^2 + (z(a) - z(b))^2\right)^{\frac{1}{2}}. \tag{7}$$

   - Any coordinate system that is related to our first by a transformation of the form

$$\mathbf{x}' = \mathbf{R}\mathbf{x} + \mathbf{a}, \tag{8}$$

     where $\mathbf{R}$ is an orthogonal 3×3 matrix and $\mathbf{a} \in \mathbb{R}^3$, is just as legitimate a way of representing the physical facts as is our first coordinate system.

(c) The generally covariant formulation

- To formulate a Euclidean-space theory generally covariantly, we need a mathematical object that is capable of representing the Euclidean spatial metric.
- A 'tensor field of type (0,2)' — a three-by-three matrix field on our 3D space, transforming in a certain way under coordinate transformations — fits the bill.
- In the 'Cartesian' coordinate systems of our standard formulation, the value of this field is everywhere the identity matrix (i.e. 'ones' on the diagonal, 'zeros' elsewhere).
  - In other coordinate systems, it is a different symmetric 3×3 matrix.
  - The *general* transformation law for $\gamma_{\mu\nu}$ is given by

$$\gamma'_{\mu\nu} = \gamma_{\alpha\beta} \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu}. \tag{9}$$

- Distances are recovered from this object γ via the formula

$$l = \int \sqrt{\gamma_{\mu\nu} dx^\mu dx^\nu}, \tag{10}$$

  in any coordinate system.
- (An exercise (straightforward), to make sure you see what's going on here: Check that if you transform γ using the rule (9) when going from a Cartesian to a spherical polar coordinate system, the value of the quantity $\gamma_{\mu\nu} dx^\mu dx^\nu$ (implicitly summing over repeated indices) is the same, for a fixed pair of 'infinitesimally separated' points, before and after the coordinate transformation.)

2. Newtonian spacetime

(a) In Newtonian spacetime N, there is a fact, for any pair of points, about their spatial distance, and another fact about the temporal displacement of the second from the first.

(b) The standard formulation

- Represent spacetime via a coordinate system $(t, x, y, z) : N \to \mathbb{R}^4$.
- Recover spatial distance facts as follows: For any two points a, b of the spacetime N, the spatial distance between a and b is given by

$$D_s(a, b) = \sqrt{(x(a) - x(b))^2 + (y(a) - y(b))^2 + (z(a) - z(b))^2}. \tag{11}$$

- Recover temporal distance facts as follows: For any two points a, b of the spacetime N, the temporal displacement $D_t(a,b)$ of b from a is given by

$$D_t(a,b) = t(b) - t(a). \tag{12}$$

- The privileged coordinate systems are related to one another by transformations of the form

$$\begin{aligned} t' &= t + \alpha \\ \mathbf{x}' &= \mathbf{x}' = \mathbf{R}\mathbf{x} + \mathbf{a}, \end{aligned} \tag{13}$$

where $\alpha \in \mathbb{R}$, and $\mathbf{R}, \mathbf{a}$ are as above.

3. Galilean spacetime

   (a) In Galilean spacetime, for any pair of points, there is a fact about their temporal distance. There is *no* fact about their spatial distance, *unless* their temporal distance happens to be zero. There *is*, however, a fact, for any line in the spacetime, about whether or not that line is straight. (If a line is straight *and* some of its points are at nonzero temporal distance from one another, we call it an *inertial* trajectory.)

   (b) The standard formulation

   - As in the Newtonian case: Represent Galilean spacetime G via a coordinate system $(t, x, y, z) : M \to \mathbb{R}^4$.
   - Recover temporal displacement facts from coordinates exactly as in the Newtonian case.
   - The spatial distance between two points a, b *that are such that* $\Delta t(a,b) = 0$ is given by the same formula as in the Newtonian case.
   - A line in M is straight (i.e. inertial, for lines that do not lie in a single simultaneity slice) iff its image in $\mathbb{R}^4$ under our coordinate mapping is straight in the usual sense in $\mathbb{R}^4$.
   - The privileged coordinate systems are related to one another by transformations of the form

$$\begin{aligned} t' &= t + \alpha \\ \mathbf{x}' &= \mathbf{x}' = \mathbf{R}\mathbf{x} + \mathbf{a} - \mathbf{v}t, \end{aligned} \tag{14}$$

   where $\mathbf{v} \in \mathbb{R}^3$, and other transformation parameters are as above.

4. Generally covariant formulations of Newtonian and Galilean spacetime

   - It is relatively complicated, mathematically, to formulate Newtonian and Galilean spacetimes in a generally covariant way. But it can be done, and involves no conceptual departure from what we have already done for linear time and Euclidean space. Those interested in the details (and able and willing to conquer the maths, i.e. basic concepts of differential geometry) can look them up in e.g. (Earman, 1989).

5. Minkowski spacetime

   (a) We have already seen the standard formulation of the theory of Minkowski spacetime: spacetime intervals are recovered from privileged (i.e. Lorentz) coordinate systems via the expression

$$d(a,b) = \sqrt{(\Delta t(a,b)^2 - \Delta x(a,b)^2 - \Delta y(a,b)^2 - \Delta z(a,b)^2)}, \quad (15)$$

   and the privileged coordinate systems are related to one another by Lorentz transformations.

   (b) The generally covariant formulation

   - Just as we represented the Euclidean spatial metric using a 3×3 matrix field (γ_{ab}) on a three-dimensional space, so we can represent the Minkowski spacetime metric using a 4×4 matrix field, usually written η_{ab}, on a four-dimensional space(-time).
   - In the privileged (i.e. Lorentz) coordinate systems, η takes the special form

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (16)$$

     at all points of spacetime.
     In other coordinate systems, η is represented via some other symmetric matrix. (Cf. the analogous points for Euclidean space.)
   - In *any* coordinate system, we can recover the 'Minkowski length' of a curve via an integral along that curve:

$$d(a,b) = \int_a^b \sqrt{\eta_{\mu\nu} dx^\mu dx^\nu}. \quad (17)$$

6. Special-relativistic dynamical theories

   (a) Above, we merely looked at the theory of Minkowski spacetime *itself*. What we are really interested in are dynamical theories (e.g., electromagnetism) that we might formulate in the setting of Minkowski spacetime. Each of these dynamical theories, too, can be formulated in the standard and in the generally covariant way.

   (b) Standard formulation of a special-relativistic dynamical theory

   - Example 1: Law of inertia. As before, this takes the form

$$\ddot{\mathbf{x}} = 0 \quad (18)$$

     (valid only in preferred coordinate systems).

- Example 2: Maxwell's equations. In the standard formulation, these take the familiar form

$$\frac{\partial F_{\mu\nu}}{\partial x^\nu} = J_\mu; \tag{19}$$

$$\frac{\partial F_{\mu\nu}}{\partial x^\sigma} + \frac{\partial F_{\nu\sigma}}{\partial x^\mu} + \frac{\partial F_{\sigma\mu}}{\partial x^\nu} = 0. \tag{20}$$

  (again, valid only in the preferred coordinate systems).

- If we 'accept special relativity' we require dynamical laws to be *Lorentz covariant*: that is, we require that, if some configuration of fields and particle trajectories to spacetime satisfies our equations relative to one preferred coordinate system ('inertial frame'), then the same configuration satisfies the same equations relative to all preferred coordinate systems. (This is what we meant, above, by the requirement that 'none of the preferred [i.e. Lorentz-transform-related] coordinate systems is better than any other'.)
  - The laws in both of the above examples meet this requirement.
  - This requirement is the physical content of special relativity, in standard formulation.

(c) Generally covariant formulation of a special-relativistic dynamical theory

  i. For the generally covariant formulation of a special-relativistic dynamical theory, we write down equations coupling our theory's dynamical fields not only to each other, *but also to* η_{ab}.

  - Generally covariant form of the ;aw of inertia:

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\nu\sigma}\frac{dx^\nu}{d\lambda}\frac{dx^\sigma}{d\lambda} = 0. \tag{21}$$

  - Generally covariant form of Maxwell's equations:

$$F_{\mu\nu;\nu} \equiv \frac{\partial F_{\mu\nu}}{\partial x^\nu} - \Gamma^\lambda{}_{\mu\nu}F_{\lambda\nu} - \Gamma^\lambda_{\nu\nu}F_{\mu\lambda} \tag{22}$$

$$= J_\mu; \tag{23}$$

$$F_{[\mu\nu;\sigma]} \equiv \frac{1}{3}(\frac{\partial F_{\mu\nu}}{\partial x^\sigma} - \Gamma^\lambda{}_{\mu\sigma}F_{\lambda\nu} - \Gamma^\lambda{}_{\nu\sigma}F_{\mu\lambda} \tag{24}$$

$$+\frac{\partial F_{\nu\sigma}}{\partial x^\mu} - \Gamma^\lambda_{\nu\mu}F_{\lambda\sigma} - \Gamma^\lambda_{\sigma\mu}F_{\nu\lambda} \tag{25}$$

$$+\frac{\partial F_{\sigma\mu}}{\partial x^\nu} - \Gamma^\lambda_{\sigma\nu}F_{\lambda\mu} - \Gamma^\lambda_{\mu\nu}F_{\sigma\lambda}) \tag{26}$$

$$= 0. \tag{27}$$

  - (Here, in each case, Γ is the 'Christoffel symbol', which is a fixed function of the matrix field η_{ab}.)

ii. These equations are *generally* covariant: i.e. if some configuration of fields on and particle trajectories in spacetime satisfies the equations relative to one coordinate system, *it follows that that configuration satisfies the same equations relative to an arbitrary coordinate system.* (The transformation law for the quantities Γ is such as to make this true.)

(d) The relationship between the two formulations

i. The Minkowski tensor field η_{ab} is unusually simple, as physical fields go. This is reflected in the fact that *there are* coordinate systems in which (i) it takes a very simple value, the diagonal matrix $diag(-1, 1, 1, 1)$, and (ii) it takes that *same* simple value at every point of spacetime. (Contrast: the Maxwell-Faraday tensor F_{ab}, in an arbitrary solution to Maxwell's equations, is in general a relatively complicated matrix at any given spacetime point, and also varies from one point of spacetime to another.)

ii. This invites us to work only in the privileged coordinate systems in which the coordinate components of η_{ab} are always 0 or ± 1 and the Christoffel symbols Γ are all zero, *and to suppress explicit mention of* η *and* Γ, replacing such expressions as η_{03} with the numbers that are the values of components of η in our chosen coordinate system.

iii. The resulting set of equations is the standard formulation of the theory.

- But we now understand *why* the 'privileged' coordinate systems are privileged: Lorentz coordinate systems are privileged because we have written e.g. '-1' in place of 'η_{00}' in our dynamical equations, and it is only in Lorentz coordinate systems that those two quantities are equal.

2.4 'Interpretations' of special relativity

1. What is special relativity? Some candidate answers:

(a) Special relativity as a principle theory

i. 'Special relativity consists of the Relativity Principle, the Light Postulate, whatever supplementary principles are needed to derive the Lorentz transformations therefrom, and the said derivation of the Lorentz transformations.'

(b) Special relativity as a statement about transformations between privileged coordinate systems

i. 'Special relativity is the statement that the laws of physics (in standard formulation) are Lorentz covariant.'

(c) Special relativity as a statement about the structure of spacetime

i. ‘Special relativity is the statement that spacetime structure (over and above topological and differential structure) is exhausted by the Minkowski metric.’

2. There is probably no sensible debate about which of these statements is ‘really special relativity’.

   (a) E.g. A physicist we would ordinarily describe as ‘accepting special relativity’ probably believes all three (sets of) statements.

3 Simultaneity: relativity and conventionality

There is an extended (and still ongoing) dispute about the status of the concept of *simultaneity* in special relativity: in particular, whether or not special relativity reveals simultaneity to be a matter of *convention*, as opposed to physical fact.

Historically, the arch-conventionalists were Reichenbach (1958) and Grunbaum (1973). The one-sentence caricature of the history of the discussion is that prior to 1977, conventionalism was generally accepted, and that in 1977 the tide was turned when David Malament proved a theorem supposed to drive nails into the conventionalists’ coffin. As usual, however, the one-sentence caricature is an oversimplification. There *is* a current anti-conventionalist consensus, but it is far from universal.

3.1 Preamble 1: Simultaneity in pre-relativistic physics

1. Before the advent of relativity theory, it was assumed without question that there was a matter of fact about which events are *simultaneous with* which others.

2. Theorising about the nature of the simultaneity relation often connected simultaneity with causation.

3. Kant:

   (a) An event A is prior to event B iff A is a cause of B and B is not a cause of A.

   (b) Two events A, B are simultaneous iff neither is prior to the other, i.e. iff either

      i. Neither is a cause of the other (the ’negative causal criterion’), or

      ii. A is a cause of B and B is a cause of A (the ’positive causal criterion’).

4. In *prerelativistic* physics, we are supposed to allow causal influences to travel with any speed (perhaps including infinite speed). Then, according to Kant’s definitions, simultaneity is an equivalence relation on the class of events.

5. Special relativity and Kantian simultaneity

   (a) Given special relativity, if we hold onto Kant's definitions and we read 'is a cause of' as 'is in or on the past lightcone of', any two spacelike separated events count as simultaneous with one another (by the negative causal criterion).

      i. This is the notion that Reichenbach and Grunbaum call 'topological simultaneity'.
      ii. Simultaneity, thus defined, is not a transitive relation (hence, not an equivalence relation).
      iii. This means that it cannot correspond to the relation of membership of the same constant-t hypersurface of any global coordinate function.
      iv. This notion of 'simultaneity' is sufficiently unlike our pre-relativistic idea that we are motivated to look for better deservers of the name.

3.2 The standard ('Einstein-Poincare') account of simultaneity in special relativity

1. Einstein defined simultaneity in an inertial frame F in terms of light signals.

2. Einstein-Poincare synchrony: Let O_A, O_B be inertial worldlines that are stationary in F. Let A_1, A_3 be events on O_A, and let B_2 be an event on O_B.

   Let a light signal leave O_A at the event A_1, reaching O_B at the event B_2; let the signal then be reflected immediately back to O_A, arriving at the event A_3. Say that the clocks t_A, t_B are *Einstein-synchronous relative to the frame F* iff

   $$t_B(B_2) - t_A(A_1) = \frac{1}{2}\left(t_A(A_3) - t_A(A_1)\right) \tag{28}$$

   (intuitively: iff according to that pair of clocks, the light signal takes the same time to travel from O_A to O_B as it requires for its return from O_B to O_A).

   - Note well that *statements about one-way speeds presuppose a standard of simultaneity.*

3. Einstein-Poincare synchrony is frame-relative: events that are Einstein-synchronous relative to one frame F will not in general be Einstein-synchronous relative to a different frame F'. (See figure 3.)

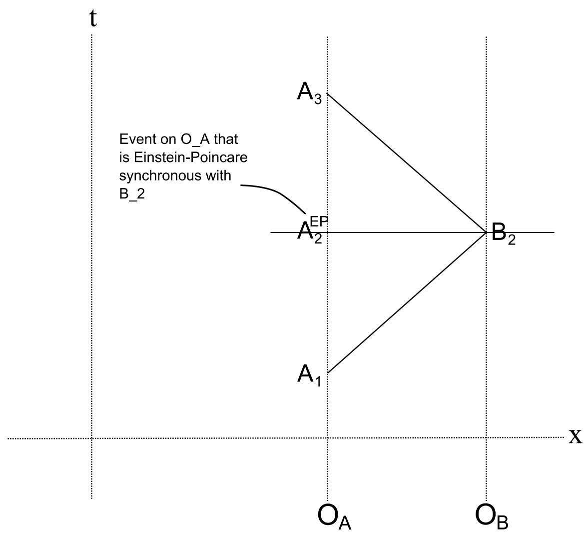

Figure 4: Einstein-Poincare synchrony. The horizontal line is the locus of points that are Einstein-Poincare simultaneous with A_2^{EP} relative to the rest frame of the observer O_A.

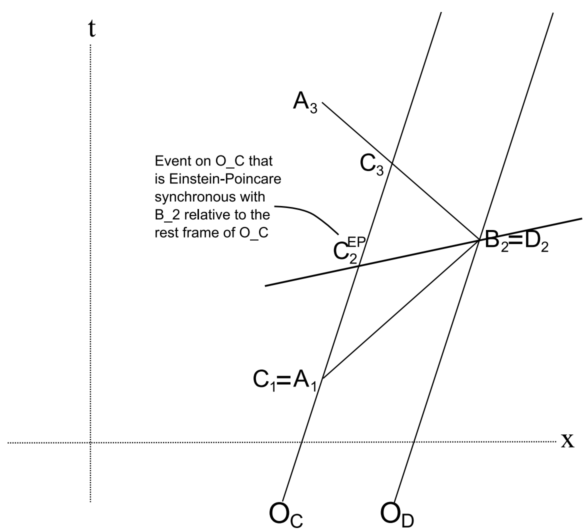

Figure 5: Einstein-Poincare synchrony relative to a 'moving' frame. Note that the locus of events that are simultaneous with the event B_2 relative to the rest frame of O_C is not the same as the locus of events that are simultaneous with the same event relative to a different inertial frame (compare the present figure with figure 4 above).

3.3 Preamble 2: Fact vs convention

1. One of the most difficult (and also the most important) tasks in the foundations of physics is distinguishing between those aspects of a given theory that should be taken to represent (or to purport to represent) aspects of physical reality, and when, on the other hand, a given apparent amendment of theory amounts merely to a change of descriptive convention.

   (a) A trivial example at each extreme

   - Example 1: Jones formulates classical mechanics using the letter x to represent a particle's spatial position. Smith uses r to represent spatial position, but otherwise her formulation of classical mechanics is identical to Jones's.
     - Clearly, Jones and Smith do not have rival theories: merely different notational conventions.
   - Example 2: Davies has a theory that predicts that a cannonball fired horizontally in a uniform gravitational field will describe a parabolic path. Evans has a theory that predicts that such a cannonball will traverse a straight-line path.
     - Clearly, Davies' and Evans' theories disagree over genuine matters of fact (since they disagree over experimental predictions).

   (b) These trivial examples help to give us an initial handle on what the distinction we're after is supposed to be, but they don't help us with 'the hard bit'; working out where, or how, we can draw the line in a principled way between those two extremes.

   (c) 'Conventionalists' about simultaneity contend that the Einsteinian account of simultaneity has the status of a convention, rather than fact, so that one could offer an alternative account of simultaneity without disagreeing with Einstein on any factual matter.

   (d) All parties agree that the Einsteinian definition is more natural than many of the suggested alternatives.

3.4 Two alternative synchrony schemata

1. Reichenbach's rule, version 1

   (a) We proceed as in Einstein's definition, but replace the factor of $\frac{1}{2}$ in equation (28) with a parameter ϵ. ϵ is then permitted to take any value in the interval $(0, 1)$.

   (b) Example: setting $\epsilon = \frac{1}{4}$ has the consequence of 'tilting' the lines of simultaneity between the worldlines O_A and O_B, as in figure 1b.

   (c) Torretti: this is not an 'inertial timescale' (Torretti, 1996, p. 225)

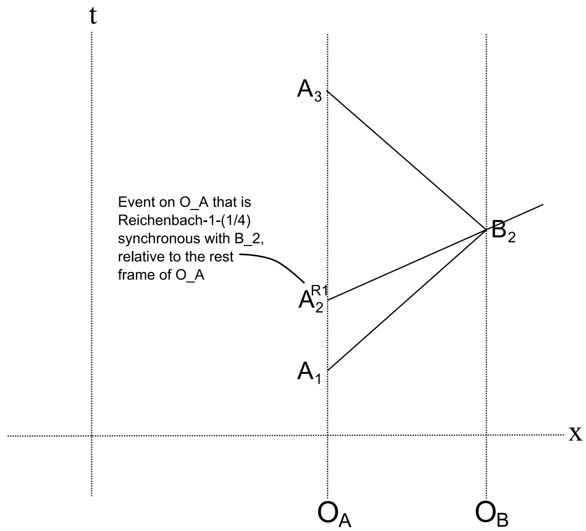

Figure 6:

   i. Torretti's point: Suppose we synchronized a clock on O_A, not only with clocks to the right of O_A, but also with clocks to the left of O_A (see figure 7), using the same (e.g. the $\epsilon = \frac{1}{4}$) synchrony rule. Then, just as the synchrony surface to the right of O_A was 'uphill heading away from O_A', so also will be the synchrony surface to the left of O_A. This means that the simultaneity surfaces are not flat. A result of *that* is that no time coordinate respecting this sort of synchrony relation can make it true that all inertial worldlines count as traversing equal distances in equal times, i.e. no such time coordinate can be an 'inertial timescale' in Torretti's (1996, p. 17) terminology. (See figure 8.)

2. Reichenbach's rule, version 2 (Torretti, 1996, pp.225-6)

   - The Reichenbach-1 synchrony scheme resulted in a non-inertial timescale because we used the *same* non-$\frac{1}{2}$ value of ϵ for every spatial direction.
   - The idea of the Reichenbach-2 scheme is to retain the feature $\epsilon \neq \frac{1}{2}$, but nevertheless to end up with an intertial timescale, by allowing ϵ to vary with spatial direction.
   - The scheme:
     - For each frame F, choose a direction $\mathbf{r}^F_{max}$ and a value $\epsilon^F_{max} \in (0, 1)$.
     - For events that are a positive distance from O_A in the spatial direction $\mathbf{r}_{max}$, apply the Reichenbach-1-(ϵ_{max}) scheme as above.

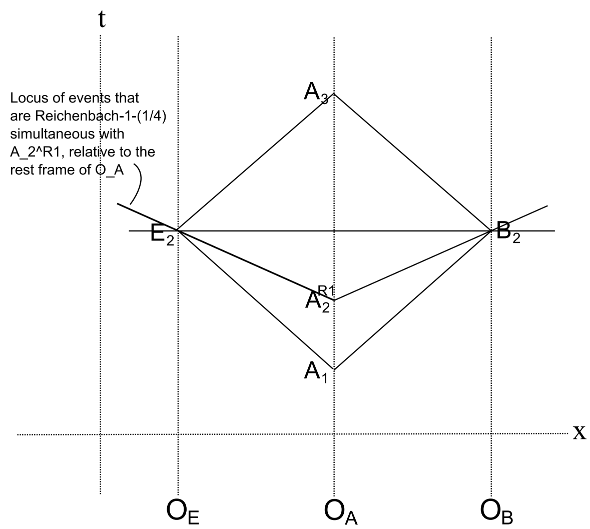

Figure 7: The surfaces of simultaneity given by the Reichenbach-1 synchrony scheme, with values of ϵ other than $\frac{1}{2}$, are not flat planes — they are hypercones, with apex on the worldline O_A relative to which the synchrony scheme has been applied.

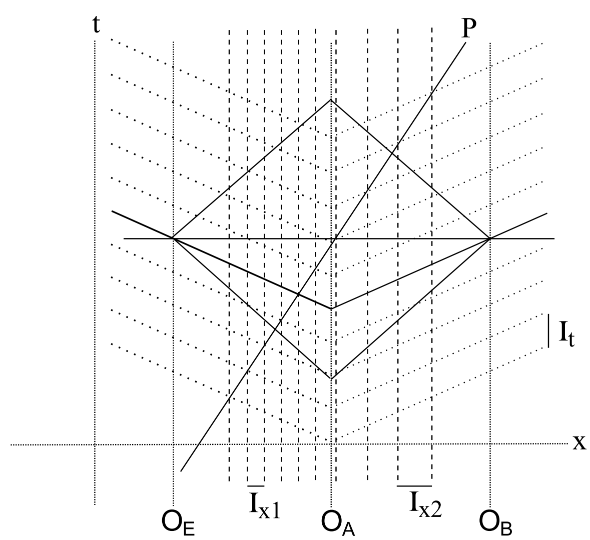

Figure 8: Relative to the Reichenbach-1-(1/4) synchrony scheme, the distance travelled by the free particle P during a *fixed* time interval I_t is smaller when it is on the left of the worldline O_A than when it is on the right of that worldline. That is, this synchrony scheme results in a non-inertial timescale.

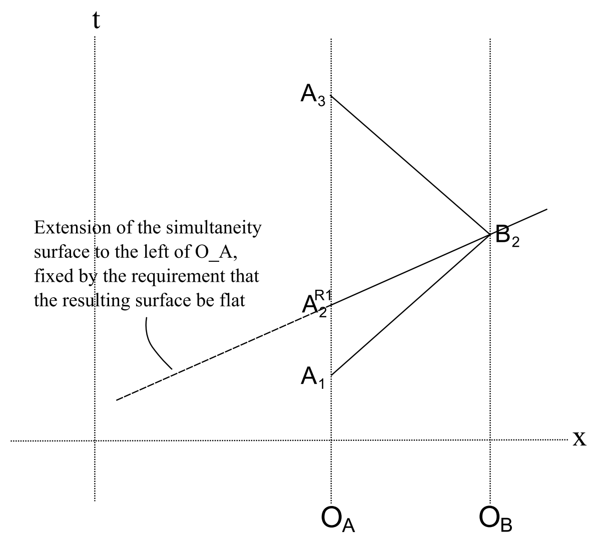

Figure 9: Unlike the 'Reichenbach-1' synchrony scheme, a Reichenbach-2 scheme defines an inertial timescale. For a fixed value of ϵ_{max}, a Reichenbach-2 synchrony scheme is relative to an inertial worldline and spatial direction.

- Extend the simultaneity surfaces to events in other spatial directions from O_A by imposing the requirement that the simultaneity surfaces be (hyper)planes, i.e. that the resulting timescale be inertial. (See figure 9.)
- The resulting simultaneity relation will be that given by Einstein synchrony for *some* inertial frame, but not necessarily the frame F whose notion of simultaneity we are defining.

3.5 Synchrony by slow clock transport

1. If the world is in fact (even approximately) special-relativistic, and some careful definition is involved in setting up a standard of synchrony of a special-relativistic world, the question is raised of how we have in fact established (even approximate) standards of synchrony, even for practical purposes, prior to explicit consideration of these issues.

2. The answer is (usually) *clock transport synchrony*: we synchronise clocks at a common spatiotemporal location, then move the clocks apart, *and continue to regard them as telling the same time as one another as they are moved apart.*

3. Special relativity calls the coherence of this practice into question: In general, two clocks that are synchronised with one another at a common point of spacetime, are moved spatially away from one another and then

reunited, *will not agree* (according to SR) about the amount of time that has elapsed since they were synchronised. We therefore *cannot* regard them *both* as 'telling the right time' (or: defining the temporal coordinate) at the point at which they are reunited, on pain of contradiction.

4. However, even in special relativity, *in the limit in which the speed of the clocks (better: the ratio of the speed of the clocks to the speed of light) tends to zero*, their disagreement on being reunited tends to zero also.

5. Since, pre-relativistically, we only ever moved clocks at speeds a very small fraction of the speed of light, this explains both (a) why we do not notice the discrepancy in everyday life, and (b) why (slow) clock transport is a perfectly adequate synchrony scheme for practical purposes.

    (a) Aside: Once we know where to look, it does not require immense technological capability to observe the phenomenon of disagreement on reunion. (The Pan Am experiment.)

6. It can be shown (Eddington, 1924) that, in the limit $\frac{v}{c} \to 0$, synchrony by slow clock transport leads to the same standard of simultaneity as does the Einstein-Poincare convention.

    (a) Recall that this standard is frame-relative. This is unsurprising, since what counts as 'slow' clock transport is also frame-relative.

7. A residual worry: What counts as 'slow' is not only relative to 'frame' in the sense of 'standard of rest', but also to *standard of synchrony*, i.e. to the very thing that we are trying to define. So isn't the procedure of synchrony by slow clock transport viciously circular?

8. Solution to this worry (Bridgman, 1961, p. 65): use the 'self-measured' clock speed (i.e. the ratio of (i) distance travelled according to the frame for which we are setting up a standard of synchrony to (ii) journey time according to the clock itself.)

    (a) The limit 'self-measured speed tends to zero' is the same limit as the (initially undefined) limit 'speed in the frame in question tends to zero'.

3.6 A brief history of the debate over the conventionality of simultaneity, post-Einstein

3.6.1 The original conventionalists: Reichenbach and Grunbaum

1. R and G claim that the only nonconventional basis for claiming that two distinct events are not simultaneous would be the possibility of a causal influence connecting the events. Accordingly, they claim that in SR, extra-convention facts do not suffice to fix the simultaneity relation — an element of conventionality is present.

2. 'Metrical simultaneity' (as opposed to 'topological simultaneity'):

   - Recall that, in SR, 'topological simultaneity' is not an equivalence relation. We *require* metrical simultaneity to be an equivalence relation.
   - We further require the metrical simultaneity relation to have the feature that if two events are metrically simultaneous, then they are also topologically simultaneous.
   - But this desideratum does not fix the metrical simultaneity relation uniquely.
   - Einstein's definition provides one way of fixing a metrical simultaneity relation. Reichenbach's and Grunbaum's point is that (as shown by the Reichenbach-1 and Reichenbach-2 synchrony schemata outlined above) there are also other ways of doing it, consistent with the only desiderata we have seen so far.

3. Reichenbach's conventionality argument

   - Reichenbach (at the stage of his career that we are currently interested in) signs up to broadly logical positivist doctrines. The key for our purposes is the *verificationist theory of meaning*, according to which the meaning of a sentence is identified with the conditions of its verification. This theory of meaning has the consequence that if 'two' scientific theories are empirically equivalent (i.e. make all the same predictions for outcomes of experiments as one another), then they *mean the same thing as* one another (in this sense, they are more properly described as two formulations of 'the same theory'). In this case, while we might *prefer* one formulation to the other if it offers us a simpler way of describing the facts, there cannot be any question about which of our theories is *more likely to be true*. The choice between such theories is merely a choice of description, i.e. is a choice of convention.
   - Accordingly, the key for Reichenbach is whether or not changing the simultaneity relation leads to an empirically inequivalent theory; his key observation is that it does not, and from this he concludes that simultaneity in special relativity is conventional.
   - Summing this up: Reichenbach's argument seems to be:

     P1: Empirical equivalence. Versions of SR that differ only on the standard of simultaneity are empirically equivalent.

     P2: Criterion of conventionality. If two sets of statements are empirically equivalent, then they agree on all matters of fact, and the choice between them is a choice of convention.

     Conclusion: Conventionality of simultaneity. The choice between versions of SR that differ only on standard of simultaneity is a choice of convention.

4. Grunbaum's conventionality argument

   (a) Grunbaum rejects positivism and the verificationist theory of meaning, and seeks to offer an argument for the conventionality thesis that does not rely on it.

   (b) Grunbaum has a notion of the *primitive quantities* that exist according to the theory. He *asserts* that these include the topological structure of spacetime, and the causal structure.

   (c) Other quantities (*including* metrical structure, and simultaneity) are taken to be factual (as opposed to conventional) iff they are definable in terms of these primitive quantities.

   (d) Grunbaum asserts that no simultaneity relation is definable in terms of the causal structure. He concludes that simultaneity is conventional.

   (e) Summing this up: Grunbaum's argument seems to be:

      P1: Basic quantities. The basic spatiotemporal quantities are the topology of the spacetime manifold, and the facts about which pairs of spacetime points are causally connectible.

      P2: Criterion of factuality. A spatiotemporal quantity is factual iff it is definable in terms of the basic quantities; otherwise it is conventional.

      P3: Indefinability of simultaneity. Simultaneity is not definable in terms of topology and causal-connectibility facts.

      Conclusion: Conventionality of simultaneity. Simultaneity is conventional, not factual.

5. Neither Reichenbach's argument nor Grunbaum's is a good argument.

   - Reichenbach's (positivist) criterion of conventionality counts too many things to be choices of convention, not fact. For instance, disagreements about the existence of a being something like the Judeo-Christian-Muslim God are presumably genuine disagreements, but according to positivism would be classed as pseudo-disputes ('mere choices of descriptive convention').
   - It's unclear what the motivation is for Grunbaum's notion of conventionality.
   - *Arguably*, Malament's theorem (below) shows that Grunbaum's third premise — his claim about the indefinability of simultaneity — is false. But this is controversial: look at the theorem and make your own mind up!

6. But bad arguments can have true conclusions, so simultaneity may yet be conventional.

3.6.2 Phenomenological counterarguments to the Reichenbach-Grunbaum conventionality thesis

1. In response to Reichenbach and Grunbaum's conventionality thesis, it was sometimes claimed that simultaneity was empirically accessible (i.e. that convention-free phenomena together with the laws of physics could establish the holding of simultaneity relations).

2. *If* this claim was true, it would show simultaneity to be non-conventional by both Reichenbach's lights and Grunbaum's.

    - The claim *isn't* true. But it's worth considering, if only because significant physical insight can be gained by identifying where each of the arguments for it fails.

3. Arguments for the claim that versions of SR that disagree only on simultaneity relation are empirically *in*equivalent

    (a) Argument from the measurability of the one-way speed of light

    - The speed of light *has been measured.*
        - Rømer 1676: delay in eclipse of one of Jupiter's moons
        - Bradley 1726: evidence for the speed of light from stellar aberration
        - Fizeau 1849: measurement of light speed via cog-and-mirror apparatus
    - More on Rømer
        - Astronomical tables suggested that Io, one of Jupiter's moons, should move into the shadow of Jupiter at a certain time. These tables were based on numerous observations of previous eclipses, from which the average orbit time etc of Io had been calculated.
        - Rømer noticed that certain irregularities in the recorded intervals eclipse times — irregularities that other astronomers had been treating as random — were actually systematic: eclipses of Io tended to be delayed (respectively, advanced) (relative to the prediction based on curve-fitting) by predictable amounts when the Earth was at points in its orbit further from (resp. closer to) Jupiter. He also noticed that this systematicity could be explained by the hypothesis that light travelled with a finite speed.
        - The accepted prediction for the eclipse on November 9, 1676, was 45 seconds after 5:32am. Rømer correctly predicted that this eclipse would occur exactly 10 minutes later than this prediction.
        - One can calculate the *magnitude* of the one-way speed of light from the *amount* of time by which eclipses are delayed/advanced.

- More on Bradley
  - If you try to catch vertically-falling rain through a straw as you are walking along (without the rain touching the insides of the straw!), you don't hold the straw vertically: you need to tilt it forwards slightly. This is because the rain falls at a finite speed.
  - Similarly, to view a star through a telescope, one cannot *quite* point the telescope straight at the star: one needs to aim the telescope slightly 'off' the line along which light is arriving, due to the fact that the Earth has (in general) a non-zero velocity in the plane perpendicular to the arriving light ray, and the fact that the speed of light is finite.
  - This effect is small, but noticeable. By measuring the 'angle of aberration' — the angle by which the telescope must be aimed 'off' in order to 'catch' light from a given star — one can, again, calculate the speed of light.
- The Fizeau measurements are of the *average round-trip* speed of light. But the Romer and Bradley measurements are of the *one-way* light speed.
- The claim (made by those who take such measurements to refute the conventionality thesis) is that: since the one-way speed of light can be measured, and is consistent with only one synchrony scheme, there can be only one synchrony scheme that is factually correct. All observations indicate that this scheme is the Einstein-Poincare one.

(b) Reply: These measurement procedures presuppose synchrony by slow clock transport. The fact that the one-way speed of light can be measured *once a particular synchrony procedure has been presupposed* is (of course) unsurprising, and (of course) shows nothing about the conventionality or otherwise of any given synchrony scheme.

(c) Synchrony presuppositions in the Romer measurements

- To calculate the one-way speed of light from the Romer measurements: Let ΔT be the time interval between two particular successive eclipses (as recorded by clocks on Earth) that we would expect on the assumption that light travelled at infinite speed. Let the measured interval between those eclipses be $\Delta T + \delta t$. Let r be the distance between the positions of the Earth when the two eclipses are observed (in, say, a frame in which the Sun is stationary; for present purposes we can regard this frame as inertial). Then, the one-way light speed is given by $\frac{r}{\delta t}$.
- But δt is the time lapse recorded by a clock that is *moving relative to the frame we are using* (viz., a frame in which the two observation events are a distance r apart). Thus, to presuppose that it records the 'true' time lapse between these events (rela-

tive to the frame in question) is to presuppose synchrony by slow clock transport.

(d) Synchrony presuppositions in the Bradley measurements

- A minute's reflection should convince you that there is no fact (hence, *a fortiori*, no fact that can be ascertained by measurement) about the'true angle' between the tangent to a given point on the surface of the Earth and a given star, except relative to a standard of synchrony. (The true angle is obtained by joining the position of the Earth at a given Earth-time to the location of the star *at the same time.*)
- Hence, there is no fact about the angle of aberration except relative to a standard of synchrony. Any experimental procedure for establishing angles of aberration *must*, as a matter of logic, be presupposing some such standard.
- Here is a sketch of one way one might establish the angles of aberration:
  - One notices that, while (roughly) the same stars are visible from a given point on the Earth's surface in summer and winter, the apparent angle between two given stars varies with time of year. One realises that one can account for this variation by hypothesising that the Earth is moving on a particular spatial orbit (in the rest frame of the fixed stars), with equal and opposite velocities in summer and winter. One then calculates the aberration angles that would enable one to account for the observed variations via the phenomenon of aberration.
- If one establishes angles of aberration in this way, one has smuggled in synchrony presuppositions in presupposing that the Earth travels with *equal speeds* at opposite points on its orbit. (Recall that in general, one-way speed phenomena are isotropic only relative to the Einstein-Poincare synchrony convention.)

(e) Argument from Maxwell's equations

- Anti-conventionalist argument: The speed of light (one-way or two-way) is entailed by the laws of physics (specifically, Maxwell's equations). In particular, those equations entail that the one-way speed of light is isotropic.
- Reply: The isotropy of the one-way speed of light *in Lorentz charts* is entailed by Maxwell's equations. Those same equations entail that the one-way speed of light is anisotropic in other charts. Of course, $\epsilon \neq \frac{1}{2}$ synchrony conventions do not result in Lorentz charts.

(f) Argument from the conservation of momentum

- Reply: Momentum is convention-dependent. The conservation of momentum holds only in Lorentz charts.

(g) Argument from clock transport

- Claim: We can discover facts about simultaneity by transporting clocks. What we discover is that the Einstein-Poincare means for establishing synchrony is correct.
- Reply: Synchrony by clock transport is just another synchrony schema (one that happens to coincide with Einstein-Poincare synchrony in the limit of slow clock transport).

4. General counterargument: all such schemes must fail

(a) We have already seen that a special-relativistic theory *can be given a generally covariant form.* Thus, the phenomena entailed by such a theory can be given a correct and consistent (if more complicated) description relative to *any* coordinate system. Coordinate systems that are obtained in part via an $\epsilon \neq \frac{1}{2}$ synchrony convention are just a special case of this. (Essentially the same argument as this is advocated by Janis, although he puts the argument a little differently: see the section on 'phenomenological counterarguments' in his SEP article.)

3.6.3 The impact of Malament's theorem

1. A preliminary consensus

(a) Circa 1977, the orthodox view was that Reichenbach and Grunbaum were correct: that simultaneity in special relativity is conventional.

2. Malament's theorem (1977)

(a) Malament proved that the Einstein simultaneity relation for a given inertial frame F is the only nontrivial equivalence relation that is definable from (a) the lightcone structure of Minkowski spacetime and (b) the frame F.

(b) Sketch of Malament's result:

- 'Causal automorphism': A map from Minkowski spacetime onto itself that preserves lightcone structure.
- 'O-causal automorphism': A map from Minkowski spacetime onto itself that both preserves lightcone structure, and takes all points on the worldline O to (possibly, but not necessarily, distinct) points on O.
- Claim (about definability): a relation [on Minkowski spacetime] is 'definable in terms of causal structure and the worldline O' iff it is invariant under all O-causal automorphisms.
- Claim: The only relation on Minkowski spacetime that is definable in terms of causal structure and a given inertial worldline O, apart from the trivial relation and the universal relation, is

the relation of Einstein-Poincare synchrony in the rest frame of O.

  - This should not be particularly surprising. A Reichenbach-2 synchrony relation, for instance, required the specification of a direction $\mathbf{r}_{max}$, and (as a result) clearly is not invariant under spatial rotations about the worldline O (and such rotations are of course O-causal automorphisms).

(c) As we saw above, some such definability claim was an essential premise for *Grunbaum's* argument for the conventionality thesis. Malament concludes that Grunbaum's argument is unsound on technical grounds.

  - But notice that even the standard simultaneity relation isn't defined from the primitive stuff (lightcone structure) *alone* — it's defined from lightcone structure *plus standard of rest*. It's therefore unclear why our'Reichenbach-2' simultaneity relations, which are definable from lightcone structure together with standard of rest *and specification of a spatial direction*, would be thought to be on conceptually such a very different footing from the standard relation.

3. The current consensus: simultaneity in special relativity is *not* conventional.

4. In the modern debate: it is not entirely clear what is *meant* by the claim that simultaneity isn't (or is) conventional; in particular, it is not clear whether parties on 'opposing sides' of this debate mean by 'conventional' the same thing as one another. This debate is a prime candidate for resolution by clarification of terms (in this case, the term 'convention').

4 The twins paradox

1. The (would-be) paradox

   (a) Consider two twins: One who journeys from Earth to Mars and back in a high-speed rocket, and one who stays at home. Suppose each carries a clock. According to special relativity, a clock that is moving in any given inertial frame runs slow according to that frame. Hence, according to the stay-at-home-twin's clock, the rocket twin's clock will run slow throughout its journey; the stay-at-home twin should therefore predict that his twin's clock will show less elapsed time than will his own clock, when they are eventually reunited. But (the paradox-generating thought goes) *the situation is in all relevant respects symmetrical*: it is true at every moment in the rocket twin's rest frame that the 'stay-at-home' twin is moving, hence, the rocket twin should likewise predict that *his* twin's clock will show les elapsed time than *his* own clock when they are reunited. But the twins cannot both be correct, since their predictions now contradict one another.

2. The explanation based on acceleration
   (a) Suggested explanation: The situation is relevantly asymmetrical because the rocket twin, unlike the stay-at-home twin, necessarily undergoes *acceleration* at some point during his journey. Thus, it cannot be true at all moments that he occupies some inertial frame, and this invalidates his reasoning.
   (b) This is confused: acceleration is not required
      - Acceleration-free version of the twins paradox: Replace the rocket twin with *two* clocks, each moving inertially throughout, and passing one another at the rocket twin's 'turn-around' spacetime point, at which point they are synchronised.
3. The claim that GR is required in order to dissolve the paradox
   (a) Suggested explanation: This is indeed a paradox within special relativity. To resolve it, we need to be able to reason adequately about accelerating frames, and *that* requires general relativity.
      - This is badly confused. We have already noted that there is an acceleration-free version of the 'paradox'. But that aside, and more importantly, special relativity *can* reason correctly about accelerating frames. (Special relativity can reason correctly in terms of an *arbitrary* frame, i.e., as we have already noted, it can be given a generally covariant formulation. The key difference between special and general relativity is not which frames are permitted, but whether or not the metric is 'flat'.)
4. The comoving-frames explanation
   (a) The rocket twin's reasoning is mistaken. It is indeed true at every point on his worldline that the stay-at-home twin's clock runs slow according to the rest frame that he is currently in. But the point on the stay-at-home twin's worldline that is simultaneous with the rocket twin's turn-around point in the pre-turn frame is earlier than the point on the stay-at-home twin's worldline that is simultaneous with the rocket twin's turn-around point in the post-turn frame. Neglecting this point, the rocket twin fails to count part of the stay-at-home twin's worldline, in counting how much elapsed time the stay-at-home clock records between the twins' parting and reunion. (See figure 10.)
   (b) This explanation is correct.
5. The spacetime structure explanation: the two twins traverse curves of different proper time. Each's ageing process is a clock surveying proper time along its own worldline.

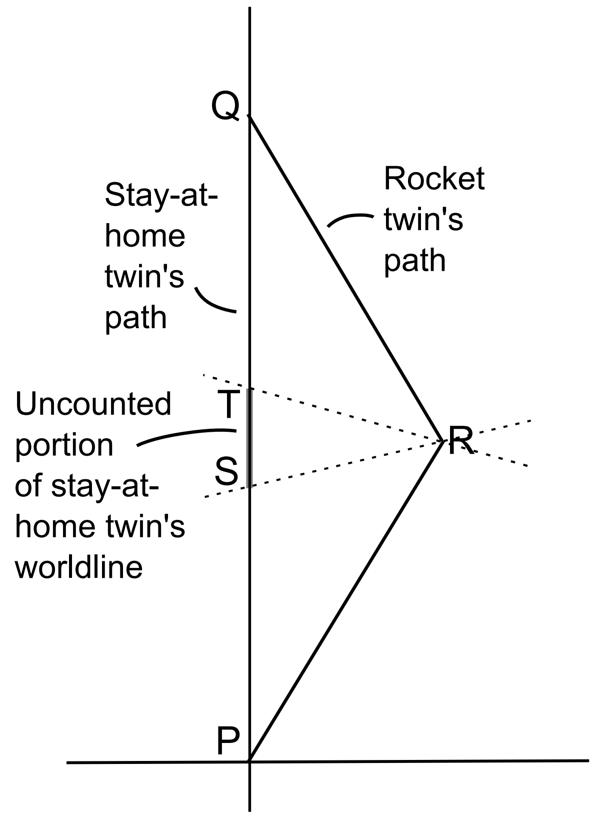

Figure 10: Diagnosis of the rocket twin's error in the twins 'paradox'. In this diagram, S is simultaneous with R in the rest frame of PR, and T is simultaneous with R in the rest frame of RQ. The rocket twin correctly reasons that the elapsed time along PS is less than that along PR, and that the elapsed time along TQ is less than that along RQ. His implicit mistake is to take the elapsed time along PQ to be the sum of that along PS and that along TQ, thus failing to count the middle portion (ST) of the stay-at-home twin's worldline.

(a) Elementary calculations show that the integral of proper time along the rocket twin's path,

$$\int_{\text{rocket path}} d\lambda \sqrt{\eta_{\mu\nu} dx^\mu dx^\nu}, \tag{29}$$

is less than the integral of proper time along the stay-at-home twin's path,

$$\int_{\text{home path}} d\lambda \sqrt{\eta_{\mu\nu} dx^\mu dx^\nu}. \tag{30}$$

Each twin should calculate the time each clock will record by integrating proper time along the relevant worldline. The result just is that the stay-at-home twin's clock will record more elapsed time (in Minkowski spacetime, a spacetime path composed of two successive future-directed timelike worldlines is shorter than the straight (timelike) spacetime path that directly joins its endpoints).

(b) Demystifying analogy: The 'wheel paradox'

- Suppose you and I live in a Euclidean space. We start at a common point of space, and we each walk in a straight line away from this point. Each of us uses a coordinate system such that we are walking in our own positive z-direction.
  - I report that you are travelling further for each unit gain in z-distance than I am. You report that I am travelling further for each unit gain in z-distance than you are.
- After some distance, I turn through 90 degrees and walk until my path once again intersects yours. As I turn, I change the coordinate system I am using, so that it will still be the case after my turn that I am walking in my own positive z-direction.
- When our paths intersect for a second time, I will have walked further than you. This can be verified by e.g. having had us each roll a wheel of the same diameter along our path, and count the number of times the wheel turns.
  - You could explain this fact by noting that throughout your journey, I was walking further per unit gain in z-distance than you were.
  - But (the 'wheel paradox') couldn't I say the same about you?
- The 'adapted frames explanation': In order to make it the case throughout my journey that I was walking along my own positive z-axis, I had to change coordinate system midway *in such a way that* the point on your path that counted as 'at the same value of z as me' *before* my change of coordinate system was further from the starting point than the point on your path that counted as 'at the same value of z as me' *after* my change of coordinate system. Hence, in my argument for the claim that you must

have walked further than me, I was double-counting part of your path. In contrast, since you used the same coordinate system throughout, you were making no such blunder.

- The space-structure explanation in this case: In a Euclidean space, an out-and-back path (two sides of a triangle) is longer than the straight path that joins its ends (the third side of that triangle). And the wheels are measuring path length.
  - It is no more mysterious than this that, in Minkowski spacetime, an out-and-back timelike path (the worldline of the travelling twin) is shorter than the straight timelike path that joins its ends (the worldline of the stay-at-home twin).

(c) The spacetime-structure is also correct, although less explanatory (since the original paradox was framed in comoving-frames language, and the spacetime-structure analysis doesn't engage with the mistaken reasoning and attempt to point out the mistake).

5 Length contraction

A thoughtful study of Minkowski diagrams shows that a given object's length (the spatial distance between its ends) is longer in its own rest frame than in any other frame.

1. Referring to figure 11:

   (a) The length of the rod in its own rest frame is $\Delta x'(a,c)$.

   (b) The length of the rod in our 'stationary' frame is $\Delta x(a,b)$.

   (c) But $\Delta x'(a,c) = \Delta x'(a,b)$. (The spatial distance between an event on the left-hand end of a rod and an event on its right-hand end *in the rod's own rest frame* is independent of which particular events on the rod-ends' worldlines we pick. You can easily convince yourself of this by drawing a Minkowski diagram of a rod that is at rest in the frame of the diagram.)

   (d) By definition (of 'length contraction'), we have length contraction iff $\Delta x(a,b) < \Delta x'(a,c)$, i.e. iff $\Delta x(a,b) < \Delta x'(a,b)$.

   (e) The Lorentz transformations tell us that (if we are using Lorentz charts) this is indeed the case: the transformation for the spatial coordinate is

   $$\mathbf{x}' = \gamma(x - vt), \tag{31}$$

   hence

   $$\begin{aligned} \Delta x'(a,b) &= \gamma\left(\Delta x(a,b) - v\Delta t(a,b)\right) && (32)\\ &= \gamma \Delta x(a,b) \text{since} \Delta t(a,b) = 0 && (33)\\ &> \Delta x(a,b) \text{since} \gamma > 1. && (34) \end{aligned}$$

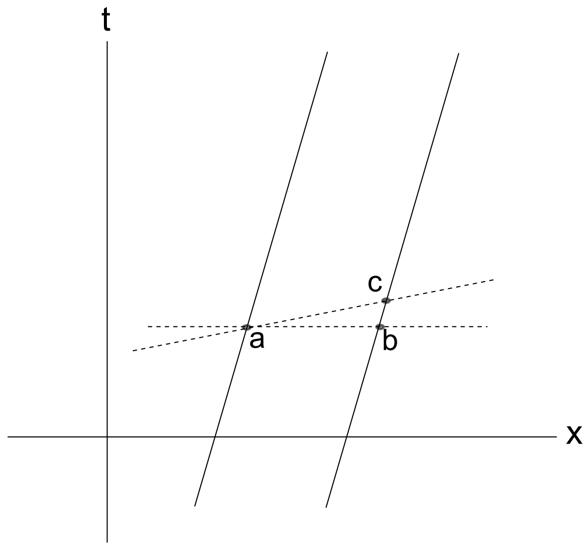

Figure 11: Length contraction. The diagram depicts the worldlines of the two ends of a rod that is moving relative to the (t, x) frame. We will have length contraction iff $\Delta x(a, b) < \Delta x'(a, c)$, where x' is the spatial coordinate in the Lorentz rest frame of the rod.

2. We can sharpen our understanding of what is going on in length contraction by making sure we have a clear account of what goes on in the following two 'puzzle cases': the 'car in a garage' problem and Bell's two-rockets puzzle.

3. Car in a garage

   (a) You have a fancy new car. It is 5m long. Unfortunately, your garage is only 4m long. It seems you have a parking problem. But then, having learnt special relativity, you have a brainwave: if you drive the car into the garage *fast enough*, you've been taught, the car will contract lengthwise. If you then slam on the brakes *really hard* and have a friend close the garage door *really fast*, you'll be able to shut the door with the car inside the garage [and worry later about how to get yourself out of the garage].

   (b) Q: Will this work? What *exactly* will happen if you try it (permitting the technologically unfeasible, but not the physically impossible)?

4. Comoving-frames analysis of the garage problem

   (a) There are various possible outcomes. Unless the car gets crushed to less than its manufactured length between the garage door and wall after having been shut in the garage, it *must* be 5m long in the rest

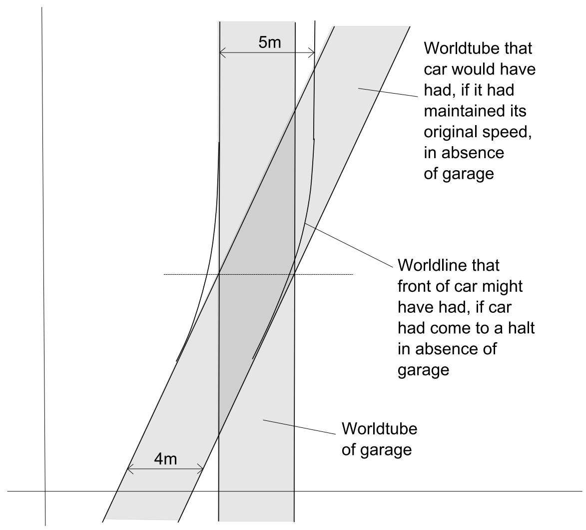

Figure 12: The garage problem. The moving car is 4m long (in the garage's rest frame). When it is brought to a standstill (in the garage frame), it must either regain its 'rest length' of 5m (in that frame), or be crushed to maintain 4m length despite its deceleration.

frame of the garage after having come to a standstill in that frame. One way to reconcile this with the fact that the car is 4m long in that same rest frame while moving is to hypothesise that the back of the car decelerates more quickly (again, in the garage's rest frame) than does the front of the car.

(b) You *could*, in principle, shut the garage door with the car inside. But a very short time afterwards, either the front or the back of the car (or both) would burst out of the garage, or (if the garage walls and door were strong enough) the car would be deformed — roughly the way you would expect a 5m long car crushed to 4m length in a vice to be deformed — in order to remain inside the garage.

5. Two-rockets puzzle

   Three small spaceships, A, B and C, drift freely in a region of space remote from other matter, without rotation and relative motion, with B and C equidistant from A.

   On reception of a signal from A the motors of B and C are ignited and they accelerate gently.

   Let the ships B and C be identical, and have identical acceleration programmes. Then (as reckoned by the observer in A) they will have at every moment the same velocity, and so remain displaced one from the other by a fixed distance. Suppose that a fragile thread is tied initially between projections from B and C[, and that] t is just long enough to span the required distance initially. (Bell, 1987, p. 67)

   (a) Q: Will the string break, as the rockets speed up?

6. Comoving frames analysis of the rockets problem

   (a) Let l be the original length of the string, in the original rest frame of the rockets.
   (b) After being boosted, the string must have *natural* length l in its new rest frame, i.e. this is the length the string *would* have in its rest frame if its length were not constrained by the fact that the rockets are artificially kept a fixed distance apart.
   (c) So what we have to work out is: whether the separation of the rockets (and hence the string's *actual*, as opposed to natural, length), in the rockets' rest frame after being boosted, is smaller than, greater than or equal to l.
   (d) From the Lorentz transformations, as above, we can deduce that it is greater than l (see figure 13).
   (e) It follows that the string will be stretched (and presumably will eventually break).
   (f) Exercise: Suppose we try to describe this scenario in the rockets' post-boost frame throughout. What is wrong with the following line of reasoning? "In the post-boost frame, the natural length of the string *increases* as a result of the boost. Hence, the string will not break; it will, rather, go slack."

6 Bell's 'Lorentzian pedagogy'

(This section is an outline of Bell's (1987) paper.)

Bell's central point: while one *can* explain phenomena such as length contraction and time dilation via comoving-frames accounts, it is not *necessary* to switch between frames in order to see what will happen in such puzzle cases in special relativity. A correct and comprehensible story can always be told from within a single frame.

1. If a string (or anything else) contracts when set in motion, this must follow from the *dynamics* that govern it. So we can predict the string's behaviour in the same way in which we predict the behaviour of anything else in physics: write down some laws and initial conditions, and see what follows.

2. Case study: An electron orbiting a moving proton

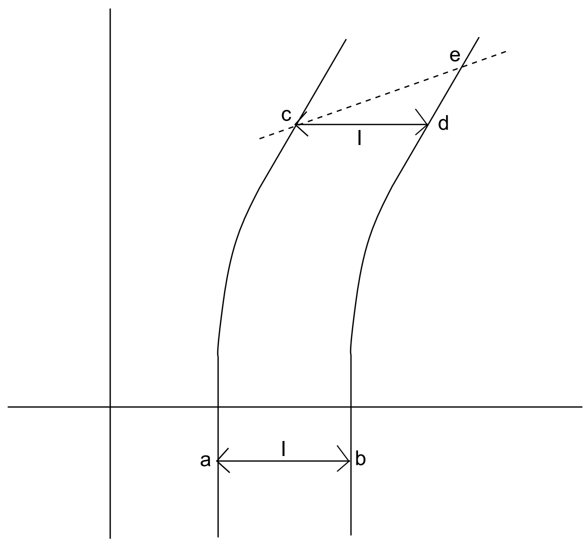

Figure 13: Bell's two-rockets problem. The rockets' acceleration programs are arranged so that they are always a distance l apart in the frame of the diagram (i.e. the frame in which they are *initially* at rest). From the fact that $\Delta x(c, d) = l$, we deduce that $\Delta x'(c, e) > l$, from which it follows (reasoning in the rockets' new rest frame) that the string is being stretched beyond its natural length.

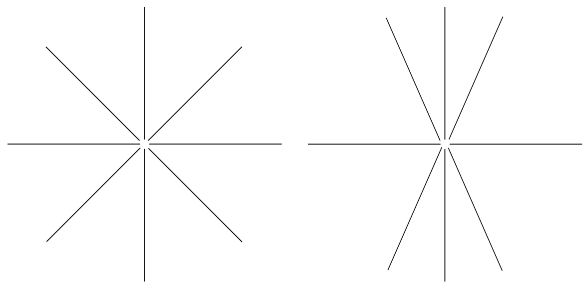

Figure 14: The left-hand diagram illustrates the spherically symmetric electric field generated by a stationary point charge. The right-hand diagram corresponds to a charge moving in the left-right direction: the lines of electric force are 'squeezed' away from the line of motion.

(a) We know, from electromagnetism, the electric and magnetic fields that are generated by a moving charge:

$$\begin{array}{rcl} E_z & = & Zez'\left(x^2+y^2+z'^2\right)^{-\frac{3}{2}} \\ E_x & = & Zex\left(x^2+y^2+z'^2\right)^{-\frac{3}{2}} \\ E_y & = & Zey\left(x^2+y^2+z'^2\right)^{-\frac{3}{2}} \\ B_x & = & -\left(\frac{V}{c}\right)E_y \\ B_y & = & -\left(\frac{V}{c}\right)E_x, \end{array} \tag{35}$$

where $z' := (z - z_N(t))\left(1 - \frac{V^2}{c^2}\right)^{-\frac{1}{2}}$.

(b) In the special case $V = 0$, these fields are (of course) spherically symmetrical. But for $V \neq 0$, *they are not.* (See figure 14.)

(c) We should therefore *expect*, on theoretical grounds, that matter in rapid motion will change shape.

3. Consider now an electron orbiting a moving nucleus.

(a) The nucleus (since it has a net positive charge) generates fields as described above.

(b) The equation of motion for an electron moving in an external electromagnetic field is given by

$$\frac{d\mathbf{p}}{dt} = -e\left(\mathbf{E} + \frac{\dot{\mathbf{r}_e}}{c} \times \mathbf{B}\right), \tag{36}$$

where $\dot{\mathbf{r}_e} = \frac{\mathbf{p}}{\sqrt{m^2 + \frac{\mathbf{p}^2}{c^2}}}$.

(c) It follows that (if the nucleus is accelerated gradually enough not to e.g. tear apart the atom) the initially circular orbit deforms into an ellipse. (This should be unsurprising, in the light of the above expressions for electric field.)

(d) Also: If the period of the orbit when the nucleus is stationary is T, it follows from the above equations of motion that the period of orbit around the moving nucleus is $T\left(1-\frac{V^2}{c^2}\right)$.

4. A change of variables:

(a) Consider the following change of variables:

$$\begin{aligned} z' &= \left(1-\tfrac{V(t)^2}{c^2}\right)^{-\frac{1}{2}}(z-z_N(t)), \\ x' &= x, \\ y' &= y, \\ t' &= \int_0^t d\tau\sqrt{1-\tfrac{V(t)^2}{c^2}}-\tfrac{1}{c^2}V(t)z'. \end{aligned} \tag{37}$$

(b) In terms of *these* variables, the orbit is 'circular' with 'period T', and has 'constant angular velocity'. *Note that*

> *The description of the orbit of the moving atom in terms of the primed variables is identical with the description of the orbit of the stationary atom in terms of the original variables. [And T]he expression of the field of he uniformly moving charge in terms of the primed variables is identical with the expression of the field of the stationary charge in terms of the original variables.* (Bell, 1987, p.72; emphasis in original)

5. Moving observers

(a) Above, we introduced the 'primed' coordinates x', y', z', t' merely for mathematical convenience, without any suggestion that e.g. t' was a 'time' coordinate.

(b) However, it is easy to see that these primed coordinates '*are precisely those which would naturally be adopted by an observer moving with constant velocity who imagines herself to be at rest*' (Bell, *ibid.*, p.75; emphasis in original).

(c) If we regard our original ('stationary') observer as being 'really' at rest, we will regard the moving observer as subject to certain systematic illusions:

- Her measuring rods are contracted in the z direction. But she doesn't realise this, because e.g. the retinas of her eyes are contracted in the z direction also.
- Her clocks run slow. But she doesn't realise this, because e.g. her thinking runs slow too.

(d) Nothing in the experimental basis for special relativity *forces* us to give up the idea that there is ('really') a standard of absolute rest. But the implications of the theory for experiment do force us to give up, once and for all, the idea that we could ever detect our absolute velocity.

6. Generalising the lesson: Lorentz covariance

   (a) Above, we proceeded by studying the specific dynamical laws (viz., Maxwell's equations and the relativistic Lorentz force law) for the phenomenon we were interested in.

   (b) But (*almost*) the only feature of these laws that we actually needed, in order to see that moving objects behave the same way in terms of the 'primed' coordinates (37) as stationary objects behave in terms of the 'unprimed' coordinates, was their *Lorentz covariance.*

   (c) 'Law L is Lorentz covariant': If we replace both x, y, z, t and the other dynamical quantities (e.g. $\mathbf{E}, \mathbf{B}, \mathbf{p}$) in law L with their 'primed' counterparts, and then we eliminate the primes using the expressions for the Lorentz transformations that relate primed to unprimed quantities, we recover the same laws we started with.

   (d) The Lorentz covariance of the laws entails a *theorem of corresponding states*: For any solution of the dynamical equations that is expressed in terms of the original coordinates x, y, z, t, one can construct a new solution by putting primes on all the variables and then eliminating these primes by means of the expressions relating primed to unprimed quantities. I.e. 'Given any state of motion of the system, there is a corresponding 'primed' state which is in overall motion with respect to the original[. And it follows from the form of the Lorentz transformations that this primed counterpart] shows the Fitzgerald contraction, and the Larmor dilation.' (Bell, *ibid.*, p.73)

7. What follows from all this?

   (a) *Not* that there is a standard of absolute rest. (The 'Lorentzian philosophy')

   (b) *Not* that one *cannot* predict, or that one cannot explain, length contraction and time dilation by first considering the description of each object in its own rest frame and using the Lorentz transformations to work out deritatively how that object will appear to observers in other frames.

   (c) Rather, that (as advertised at the outset) it is always *possible* to tell a correct and comprehensible story from within a single frame. (The 'Lorentzian pedagogy')

7 Spacetime geometry and explanation

7.1 Three explanations of length contraction and time dilation

1. We have seen three styles of 'explanation' of length contraction and time dilation
   (a) Frame-dependently, always using comoving frames
   (b) Lorentz-pedagogically: on the basis of the details of a particular Lorentz-covariant dynamics (e.g. (Bell, 1987))
   (c) Truncated-Lorentz-pedagogically: on the basis of Lorentz covariance of the dynamical laws alone
2. More on the truncated Lorentzian pedagogy: two notes of caution
   (a) As Bell noted (but did not particularly emphasise), Lorentz covariance *alone* does not entail that (hence, does not explain why) either
      - A given system in a given state *does go into* the corresponding 'primed' state when the system is boosted.
         - It *won't*, always. E.g. in Bell's example (and as he points out), if the neutron is accelerated too rapidly, the electron will not follow along and take up the Fitzgerald-contracted orbit around the moving nucleus, but will rather be left behind altogether.
         - What we hope for, and generally have, *but what is not guaranteed by Lorentz covariance*, is that if the system is boosted 'sufficiently gradually' then it will end up in the corresponding state.
            * This is the feature that HRB [earlier in this course] called 'the boostability of rods and clocks'.
            * Bell sketched how this 'boostability' may be shown for the particular system, with the particular (Lorentz-covariant) dynamics he considered by way of example, but he did not identify (and neither will we) the *general* features of his example from which it follows.
      - There are any systems that render 'length/time in frame F according to the Minkowski metric' fairly directly empirically accessible in the first place. (The *existence* of rods and clocks.)
         - Bell exhibits a system that is possible according to electromagnetism and that functions both as a natural clock and as a natural measuring rod (relative to the Minkowski metric, in the original frame). But this exhibition utilised specific details of Maxwell theory. We have no proof (and it is easy to show that it is not true) that there will be any natural rods/clocks in an *arbitrary* Lorentz-covariant theory.

(b) Therefore, to be a sound mode of explanation of length contraction and time dilation, the truncated Lorentzian pedagogy must supplement the postulate that

- the laws (whatever their details) are Lorentz covariant

with the additional postulates that the laws (whatever their details) are such that

- there exist systems that function as natural rods and clocks in the original frame, and
- the system functioning as rod/clock does indeed go into its 'corresponding state' when boosted sufficiently gently.

7.2 Explanation and spacetime realism

References:

- H. Brown (2005), esp. chapter 8.
- Brad Skow's review of HRB's book: (?, ?) available online at http://ndpr.nd.edu/review.cfm?id=6603.

This last section of the course focusses on an issue that is currently under active dispute in the foundations of physics community. The issue concerns the status of the Minkowski metric in special relativity: specifically, whether it is an independent element of reality, with ontological status on a par with other equally fundamental physical fields (e.g. the electromagnetic field), or whether, in some sense, talk of the Minkowski metric says nothing 'over and above' things that we can already say by talking about other fields, and the Lorentz covariance of the equations that govern them.

Harvey Brown's book defends a more deflationary account of the Minkowski metric than the current orthodoxy in the foundations of physics community; Skow's review indicates how a defender of the orthodoxy is likely to respond.

1. There are two (related) sets of disputed questions:

   (a) Does postulating Minkowski geometry for spacetime *explain*

   - the Lorentz invariance of the dynamical laws?
   - Phenomena such as length contraction and time dilation?

   (b) Insofar as special relativity is empirically adequate, should this lead us to believe in Minkowski geometry as an *independent* real feature of the world?

2. Two views of Minkowski geometry in answer to these questions: Insofar as we believe Special Relativity ...

   (a) 'Explanationism': ... we should believe in an independent Minkowski geometry, and (perhaps: precisely because) postulating this geometry enables us to explain various things that we can't otherwise explain.

(b) 'Codificationism': . . . we should believe that the geometry of spacetime is Minkowskian, but this latter statement is a mere codification of certain facts about the [standard-formulation] dynamical laws (e.g. their Lorentz covariance). As such, it cannot *explain* those facts.

   - 'Opium sends people to sleep because it has a dormitive virtue'. This is not an explanation.

3. Adjudicating between the Explanationist and the Codificationist will require clarity on the basic points of two areas of the philosophy of science: scientific realism, and inference to the best explanation (IBE).

4. Scientific realism vs. antirealism

   - Scientific realism (roughly): The empirical success of a scientific theory gives us a good reason to believe in the entities postulated by that theory/to believe that the theory is (approximately) true.
   - Scientific antirealism (roughly): The empirical success of a scientific theory gives us a good reason to believe *that that theory will continue to be empirically successful*, but not that what it says about goings-on beyond the observable level is true, or approximately true, or that (anything like) the unobservable entities postulated by the theory exist.
     - Examples of scientific antirealism
       * An antirealist about electrons would have no quarrel with scientists uttering sentences like 'the accelerator gives electrons an energy of X MeV', but would insist that such talk is merely shorthand for a long conjunction of statements about correlations between observables ('if I was to press the red button and place a sample of material Y in the centre of the accelerator tube then my displays would read such-and-such', and so forth).
       * Copernicus's then-controversial tome setting out heliocentric astronomy was preceded by a preface (now thought to have been written by his friend Osiander, but claiming to be written by Copernicus himself) 'explaining' that one needn't be worried about the implications of heliocentric astronomy for e.g. theology, because Copernicus's system was intended merely as a simple technique for calculating the apparent positions of stars and planets, not as a true description of the world.
       * Most of the current debate in the foundations of quantum mechanics makes sense only on the assumption that physics gives us a handle on the reality behind the observed phenomena, rather than merely a way of predicting the phenomena. Antirealists, rejecting that background assumption, regard such debates as fundamentally misguided.

  - It is essential to distinguish between antirealism *across the board* — the claim that science *in general* says nothing about the reality behind the appearances — and antirealism *about particular entities*.
  - The codificationist is *not* an antirealist across the board. He is *generally* a scientific realist, but thinks that there are special reasons for not regarding the *Minkowski metric* as an independently existing entity.
  - It is important to bear this in mind, because it means that an argument for codificationism should not 'prove too much', in the sense of equally well implying that *all* talk of unobservable entities in physics has the status of mere codification — if it does that, it is an argument for antirealism across the board, rather than for codificationism specifically about the Minkowski metric.

5. Inference to the best explanation

   - Nobody thinks that scientific theories are *deductively proved* from experimental data.
   - Nobody thinks that scientific theories proper (as opposed to: phenomenological models) are obtained by simple *inductive generalisation* from experimental data.
   - The methodology is better described as *hypothetico-deductive*: one *postulates* or *hypothesises* a theory, deduces predictions from that theory (what one would expect to see in experiments if that theory were true), and tests those predictions against experiment. If the predictions do match experiment, this is, in some sense, a strike in favour of the theory. (Here agreement ends as to what *exactly* is going on.)
   - Scientific realists are often fans of *inference to the best explanation (IBE)*: i.e., they think that inferences of the form

     P1. We have obtained data D.

     P2. Theory T is the best available explanation of data D.

     C. Theory T is [approximately] true,

     while (of course) not deductively valid, are reasonable (i.e. that it is reasonable to assign high probability to their conclusions on the basis of their premises).
   - If one agrees that IBE is a key part of the methodology of science, the two types of 'disputed question' mentioned above become intimately linked: one will think that the claim that postulating an independently existing Minkowski metric has explanatory power is a good reason for accepting the claim that there *is* an independent Minkowski metric field.

6. We approach the debate between codificationists and explanationists by identifying four separate claims (some controversial, some not), and considering arguments for and/or against each in turn.

7. Claim 1: 'Minkowski geometry explains Lorentz covariance'

   - This is one of the explanationists' claims, denied by the codificationist.
   - It should be uncontroversial (I think!) that, in a certain sense we will make precise, Minkowski geometry *entails* Lorentz covariance. It will remain controversial whether or not the arguments demonstrating this entailment amount to genuine *explanations* (as opposed to pseudo-explanations, of 'dormitive virtue' type).
   - Making things precise: there are actually *two* claims being made here.
     - Claim 1a: The claim that there is a Minkowski metric (explicit representation of which has been suppressed in our 'standard formulation') explains the fact that our standard-formulation laws are in general *not* covariant under *non*-Lorentz transformations.
     - Claim 1b: The claim that there is *no suppressed structure other than the Minkowski metric* explains the fact that our standard-formulation laws *are* covariant under *Lorentz* transformations.

     We will outline the argument for each of these two claims in a moment.
   - The common setting for both arguments: Suppose we start from a theory in generally covariant form, and that one of the fields is the Minkowski metric field $\eta_{\mu\nu}$.
     - Example (a toy theory): $\eta_{\mu\nu}v^\mu w^\nu = 1$.
   - Fact 1: In Lorentz charts, η takes the especially simple form

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{38}$$

   - Fact 2: Lorentz charts are related to one another by Lorentz transformations.
   - Defence of claim 1a:
     - Suppose we make a 'standard formulation' version of our theory by replacing the components of η with the (common) values that they take in a particular Lorentz chart, C^*.
       * In our toy example, this gives the equation

$$-v^0 w^0 + \mathbf{v} \cdot \mathbf{w} = 1. \tag{39}$$

  - Then, in general, our equations will be false in all charts in which the components of η take different values from the values they take in C*, i.e. in all non-Lorentz charts. (Only 'in general', because if the equations happen not to use e.g. the component η_{03}, then the equations will be true also charts that differ from Lorentz charts only on the value of η_{03}. Our above example illustrates the 'general case' in which all components of η are involved in the theory's equations.)
  - If T is a non-Lorentz transformation between charts, then T(C*) is a non-Lorentz chart, hence a chart in which our equations are false.
  - So T can take us from a chart in which our equations are true to charts in which our equations are false, i.e. (by definition of 'co-variance') *our theory is not covariant under the transformation T.*

- Defence of Claim 1b:
  - Suppose that our standard-formulation does not suppress anything *other* than the Minkowski metric.
  - Then, its equations are *guaranteed* to be true in all charts in which the components of η are the same as its components in C^*, i.e. are guaranteed to be true in all Lorentz charts.
    * The only way the standard-formulation equations could *fail* to be Lorentz covariant is if their derivation (from the GC formulation) involved suppressing some field whose components vary from one Lorentz chart to another (e.g., in our toy example, the four-vector v).
  - So, there is a collection of charts (the Lorentz charts) that are related to one another by Lorentz transformations, and in all of which the equations are true: i.e., the theory is Lorentz covariant.

- Where we've got to: The hypothesis that there exists a background Minkowski metric, and that there is no other structure that has been suppressed in going from the generally covariant to the standard formulation of our theory, does *entail* the result that the standard-formulation laws will be Lorentz covariant (and should lead us to expect that they will not be covariant under an arbitrarily selected non-Lorentz transformation). But whether or not this entailment-plus-expectation amounts to an *explanation* depends on whether or not the Minkowski metric is an entity ontologically indepedent of those the standard formulation directly talks about. Codificationists insist that it is not, hence don't count the above entailment as an explanation; explanationists think that it is, and that belief in such an entity is justified by IBE.

- The explanationists' position is roughly as follows:

- *Something real* must be conceived as the cause for the preference of a Lorentz over a non-Lorentz coordinate system.
  - The existence of a (real) background Minkowski metric is a candidate for this cause. We have no other candidates.
- The explanationists' main gripe with the codificationist account is that the latter is (the explanationists think) just an 'as if' theory: matter fields and particles *behave as if* they are coupling to a background Minkowski metric, but there is no Minkowski metric in reality.
  - Consider, e.g., the theory that the world is just as the ordinary person thinks it is, but there are in reality no cats: it is just that mice, photons etc behave just *as they would if* certain regions of spacetime were worldtubes of cats. This theory is empirically equivalent to the ordinary person's theory (unless you happen to be a cat), but it's clearly a stupid theory. The moral seems to be that 'as if' theories are, *in general*, bad theories.
  - One of the explanationists' challenges to the codificationist is to explain why the codificationist account of the Minkowski metric is any better than the no-cat theory.

8. Claim 2 (accepted by all parties): Lorentz covariance does not, *by itself*, explain length contraction and time dilation.

   (a) This is just the point already made above: that the truncated Lorentzian pedagogy requires assumptions of rod/clock existence and of 'boostability' in order to explain length contraction and time dilation.

   (b) Corollary: *Minkowski background structure* (whether or not one calls such structure 'geometrical') does not, *by itself*, explain length contraction and time dilation.

9. Claim 3 (orthodoxy; denied by codificationists): Minkowski background structure, when supplemented with rod/clock existence and boostability assumptions, *does* explain length contraction and time dilation.

   - This just is the 'truncated Lorentzian pedagogy'. What the codificationist denies is that the Minkowski metric is doing *explanatory work* in this story — that an account that takes the Minkowski metric as starting point is explanatorily any better than one that simply starts form the Lorentz-covariance of the standard-formulation laws. (Cf. claim 1.)

10. Some of Brown's objections to the explanationists' position, and their standard replies

    (a) Objection 1: the explanationists' account [i.e., roughly that discussed above, under 'Claim 1'] of the relationship between Minkowski geometry and Lorentz covariance is 'wholly unclear' (Brown, *ibid.*, p.134)

i. Reply: No, it isn't...what's the problem?

(b) Objection 2: Spacetime structure in SR violates the action-reaction principle (Brown, *ibid.*, section 8.3.1)

i. Reply: The action-reaction principle is neither an [ontological] criterion of reality, nor an [epistemological] criterion of legitimate postulation.

(c) Objection 3: Geometry doesn't always explain. Why would the present case be any different? (Brown, *ibid.*, sections 8.2.1–8.2.3)

i. Short reply: Well, why would the present case be the *same*?

- Nobody was offering the argument 'the Minkowski structure is geometrical structure, therefore it is explanatory'.

11. Claim 4 (acceptance roster unknown, but I think includes HRB): Saying that the Minkowski structure *is geometrical/spatiotemporal/etc structure* does not *explain* (as opposed to codify) anything.

(a) One of the issues causing confusion between Brown and his critics, I think, is that of whether or not it is permissible to postulate that such-and-such a field represents *spacetime structure* (in specified ways).

- Analogy:
  - Q: Can I identify an arbitrary collection of particles in Newtonian mechanics, and *postulate* that they compose a *billiard ball*?
  - A: Of course not: they have to behave like a billiard ball in order to deserve the name.
  - Q: Can I *explain* why they behave like a billiard ball by saying that they are one?
  - A: Of course not. (That would be like explaining why opium makes one sleepy by saying that it has a dormitive virtue.)
- Suggested methodology: *postulate* the physical reality of certain mathematically specified structures; then *argue*, on the basis of how they behave, that they deserve certain names.
  - Brown's rhetorical question: which of the two rank-two Lorentz-signature tensor fields in Bekenstein's bimetric theory 'is geometrical'? This is either a pseudo-question, or is to be answered by looking at the dynamics (to see how each of the tensors in question couples to ordinary matter); it *isn't* to be answered by mere postulation, any more than the question of whether my favourite N particles are currently constituting a billiard ball (given their positions etc) is to be answered by mere postulation.

- But the issue addressed by Claim 4 must sharply be separated from the issue of whether or not the Lorentz covariance of the dynamical laws gives us reason to believe in *the Minkowski tensor field*, specified via its mathematical structure.

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