# In search of (spacetime) structuralism

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**Abstract**

Structuralism is supposed to be a dissolver of metaphysical pseudo-debates. This paper is a search for the thesis behind the rhetoric. Taking 'spacetime structuralism' as a case study, I identify six different theses that seem to share this name. My conclusions are largely negative: that those theses that are new are not plausible, and vice versa. The exception is structuralism as a rejection of the fundamentality of the object/property/relation framework, but in this case it is as yet unclear what the *positive* thesis might be.

## 1 Introduction: In search of structuralism

In the beginning, science and metaphysics were inseparable. Many would say that things are still that way. But against this, an increasingly common theme in recent times (e.g. (Ladyman & Ross, 2007)) has been the idea that metaphysics has lost contact with science, and so much the worse for metaphysics. The picture is one of metaphysicians hiding in their ivory towers, playing with one another elaborate and increasingly sterile word-games that have quite definite rules, but no importance. Science, meanwhile, has gone from strength to strength.

The theme has a strong and a weak form. The weaker form claims that it is through science alone that metaphysics makes contact with the total body of human knowledge, and that therefore *good* metaphysics necessarily maintains contact with science (e.g. openness to the concerns of and examples from science). The charge is that metaphysics has in recent times tended to become too insular.

The stronger form aims to formulate precise criteria that will isolate certain aspects of theorising, ripe for discarding as necessarily leading only to falsehoods or even nonsense; and the idea is that those portions of traditional metaphysics that have lost contact with science, in particular, will fall into this category. Logical positivism was a particular species of this stronger genus. Logical positivism notoriously culled too much. But modern-day 'strong' theorists claim to have identified a new species that promises to succeed where logical positivism failed; this new species is called 'structuralism'.

What then *is* structuralism? It is very far from clear, and this leads to the project of the present paper. I have told the story thus far in general terms, but clarity is often served by beginning with specific examples. Spacetime will serve as a case study.

## 2 The road to spacetime structuralism

A venerable discussion in the philosophy of space and time compares the relative merits of two views of the ontological status of those entities: *substantivalism* and *relationism*. As a first pass, substantivalists contend that 'space is a substance', while relationists hold that 'space is nothing more than a system of relations among bodies'. In the context of Newtonian mechanics, Leibniz urged relationism on grounds stemming from his Principle of Sufficient Reason: if space were a substance there would be a difference between the actual universe and a universe in which (say) all material bodies had been shifted three feet to the left, but there could be no sufficient reason for God to realise the actual universe rather than this possible alternative, and God does nothing without a sufficient reason; ergo, space is not a substance. Newton retorted that relationism cannot account for the undeniable empirical fact that the surface of the water in a spinning bucket is concave. Both parties seemed to be making forceful points, and many bells and whistles (surveyed in (Earman, 1989)) ensued down the centuries.

The understanding of space and time in physics has moved on significantly since the time of Newton. Special relativity urges us, among other things, to replace talk of 'space' and 'time' separately with talk of a unified 'spacetime'. General relativity urges, among other things, that the spatiotemporal distance relations themselves cannot be fixed *ab initio*, as part of a fixed background stage on which the dynamics of matter plays itself out, but rather that they differ from one model of the theory to another, just as the behavior of matter does.

The move to general relativity, in particular, is important for the substantivalism/relationalism debate. In a seminal paper, John Earman and John Norton (1987) presented a variant of Einstein's 'hole argument'. This variant takes a basic fact about general relativity — its diffeomorphism invariance — and argues from this fact to the metaphysical conclusion that substantivalism is false. In response to this challenge, substantivalists have offered several ways of rendering their position consistent with diffeomorphism invariance. Since none of this discussion suggested that the old question of substantivalism versus relationism was not the right question to ask, I include it in 'the traditional debate'.

The traditional debate was *originally* carried on by physicists themselves. Newton *et al* were the key players in developing physical theory, and the substantivalist-relationist literature seems originally to have existed because they found it an essential part of the process of physical theorizing to try and get clear on the metaphysics of the theory. In recent years, the division of labour has been different. Physicists have continued to develop the mathematics of spacetime theory and connect it to experiment; philosophers of physics and metaphysicians have carried on the substantivalism-relationism debate. It would not take too anti-philosophical an observer to judge that at least the *majority* of practising physicists could not care less about, indeed are barely aware of, the philosophers' machinations.

This trend, insofar as it is a genuine trend, can be interpreted in either of two ways. According to the first, all we have here is an unremarkable example of increasing specialization: in the seventeenth century the physicists, chemists, biologists, mathematicians and philosophers were all the same people, and of course now they are not, but this no more means that the philosophical debates are outmoded than the fact that algebraic geometers can be almost entirely ignorant of the latest developments in population biology, and *vice versa*, means that either is outmoded. But there is a second possibility: that the scientists have lost interest in the old philosophical question because science has 'moved on' from it in some important way that the philosophers have, to their own detriment and discredit, hitherto failed to notice.

In the past five to ten years, some philosophers of physics have increasingly been feeling the suspicion that this second possibility is the one that obtains, and the philosophical literature been a surge of papers suggesting that the traditional debate is misguided. The specific point here is sometimes supposed to be that there is a 'third way', between substantivalism and relationism, that has hitherto been missed and that is superior to each of the traditional alternatives; sometimes it is that general relativity, somehow, 'dissolves' or 'overcomes' the traditional debate. Claims in this general area often go by the name of 'structuralism'.

The present paper exists because I have some sympathy with the structuralists' general project, but also the feeling that much of the existing structuralist literature is frustratingly unclear. It is not sufficiently clear (a) what exactly the various 'structuralist' theses are supposed to be, (b) how (if at all) they are related to one another, (c) whether they are genuinely alternatives to the spectrum of positions mapped out within the confines of the traditional debate, or (d) how plausible they really are. The paper is an opinionated survey of the options; it is my attempt to clarify (a), (b) and (c), with a view to facilitating further discussion of (d).

The structure of the remainder of the paper is as follows. In section 3, I state more precisely what I will take substantivalism and relationism to be. Section 4 explains the concept of diffeomorphism invariance, the feature of general relativity that is supposed to cause all the trouble. Section 5 provides an exposition of Earman and Norton's 'hole argument' against substantivalism. Section 6 canvasses the *standard* menu of substantivalist responses to this argument; this completes my exposition of 'the traditional debate'. Sections 7 and 8 then turn to the main task of the paper: disentangling, and assessing the relevance of, as many as possible of the various claims that seem to have been touted in the name of 'structuralism'. Section 9 is the conclusion.

## 3 Substantivalism and relationism about spacetime

The full substantivalist-relationist controversy is more complicated than we need to deal with here; it will suffice to examine pared-down versions, eschewing the use of several terms whose interpretation itself raises unnecessary confusion (but cf. 'structuralism v1', in section 8). Somewhat anachronistically, therefore, let us characterize substantivalism and relationism purely in terms of ontology and ideology, as follows.

**Substantivalism and relationism in a particle theory.** Assume, first, a physics that deals only with a spacetime, and point particles. Then:

*Substantivalists* hold that spacetime points are objects. They usually add to this that distance relations are fundamentally relations between points of spacetime, that particles occupy points of spacetime, and that distance relations between particles are derivative from the more fundamental spatiotemporal relations holding between spacetime points, and occupation relations holding between material objects and spacetime points.

*Relationists* deny that there are any such objects as spacetime points. According to them, the (only) fundamental physical objects are particles.

These material objects bear (spatiotemporal) distance relations to one another. Spacetime is this system of relations.

**Substantivalism and relationism in a field theory.** As a first step in the direction of general relativity, let us now suppose that, instead of point particles, we have a theory that postulates *fields* on spacetime.

For the substantivalist, this is unproblematic. Spacetime points are still objects. Our new theory requires us to recognize a new type of property that spacetime points can possess: *field values.* A given type of field is defined by a space $T$ of possible field values; a (token) field is an assignment of an element of $T$ to each spacetime point.[1]

Relationism is, arguably, less natural in a field-theoretic context, but it can be stated: the relationist project will be to represent a field as a system of ('spatiotemporal') relations on the space $T$ of field values.

## 4 Diffeomorphism invariance

General relativity is a field theory. But then, so is electromagnetism; it is not in virtue of being a field theory that general relativity has sometimes been taken to revolutionize the substantivalism-relationism debate. The distinctive feature of general relativity is, rather, that it is diffeomorphism invariant.

The basic point here is that, in general relativity (but not in earlier spacetime theories), it is not possible *first* to lay down the spacetime structure — the 'metric' of spacetime — and only *afterwards* to 'solve the theory's dynamical equations' to find out which configurations of 'matter fields' on this spacetime are dynamically allowed. (Matter and spacetime structure interact in too intimate a way for that.) Since, in general relativity, the dynamically allowed values of the metric field at a given point in spacetime depend on the values of the matter fields at that and other points, in general relativity one has to 'solve the equations for' the metric and 'matter fields' *simulataneously.* Thus, from the point of view of the theory's mathematics,

[1] For a simple example, consider Newtonian gravitation theory. This theory represents the gravitational field by a real-valued function on spacetime, the 'gravitational potential field', from which the gravitational force on a particle of a given mass and location can be derived. In this case $T$ is just (or is isomorphic to) $\mathbb{R}$, the space of real numbers. Other fields require 'target spaces' $T$ of higher dimension and more complicated structure than that of the reals, but the features these complications introduce are irrelevant to our purposes.

the metric field is 'just another field', on a par with (say) the electromagnetic field. This has the consequence that the solution space of the theory is very large, in a sense we will now explain,

We will need some formalisation. A *spacetime theory* is a theory whose models are $n$-tuples of the form $\langle M, A_i, D_j \rangle$, where $M$ is a differentiable manifold, the $A_i$ are geometrical objects specifying 'absolute structure' of $M$ ('absolute' in the sense that they are common to all of $T$'s models), and the $D_j$ are geometrical objects on $M$ representing 'dynamical' fields (which vary from one model to another). A *diffeomorphism* is a smooth map from the manifold $M$ onto itself: $h : M \to M$. To any such map, we can associate in a natural way a transformation or 'drag-along' of geometrical objects defined on $M$: write $h * F$ for the 'drag-along' of the geometrical object $F$ by the diffeomorphism $h$. Then, we say that $h$ is a *symmetry* of the theory $T$ iff, for any model $\langle M, A_i, D_j \rangle$, the structure $\langle M, A_i, h * D_j \rangle$ is also a model of $T$. We say that a theory $T$ is *diffeomorphism invariant* iff *all* diffeomorphisms are symmetries of $T$: that is, if the class of models of $T$ is invariant under the natural action of all diffeomorphisms. The point is that if our theory has, like GR, no absolute structure, then *any* diffeomorphisms will be a symmetry of the theory. Heuristically: one can 'move the matter fields around the spacetime' in any (smooth) way that one likes, *provided* one simultaneously 'moves the metric around' in exactly the same way; for dynamical equations specify merely the *relations* between the dynamical and metric fields.

## 5 The hole argument

One can worry about diffeomorphism invariance. According to Earman and Norton, the substantivalist should indeed be worried.

Say that a physical theory is *deterministic* if the theory's laws, together with the details of a solution on some open time interval, suffice to determine the solution over the whole of spacetime. As usually understood, general relativity is deterministic. Earman and Norton argue (in the first instance) that the substantivalist is forced to say, on the contrary, that the theory is indeterministic.

The point is a simple one. Consider any model $s$ of general relativity, based on a manifold $M$. Now consider a diffeomorphism $d$ of $M$ that acts as the identity transformation on the part of $M$ earlier than some arbitrarily selected timeslice, but differs from the identity thereafter. By diffeomorphism invariance, since the original structure $s$ is a model of the theory, it follows that the diffeomorphism-deformed structure $d * s$ is also. But the original and deformed solutions, while agreeing about all field values at all points earlier than your line, will in general not agree on the values of fields at any given point futurewards of your line. That is, any diffeomorphism invariant theory is indeterministic. The indeterminism (furthermore) is rather a radical one, in two ways: there are not three, or five, but uncountably many possible futures for a given past, and (worse) it is not merely that (as in quantum mechanics) multiple possibilities are permitted, but rather that (*unlike* the indeterminism of quantum mechanics) the theory does not even assign *probabilities* to the instantiations of *any* field values at *any* future points. [2]

So substantivalism seems to be incompatible with determinism, in the context of a diffeomorphism invariant theory. A relationist, on the other hand, will not agree that the pre- and post-deformed models represent distinct physical situations, since it is only locations of, not spatiotemporal relations between, field values that will have been changed by the deformation.

One might wonder: Can the substantivalist simply bite the bullet? In response to this, Earman and Norton stress that their point is not that determinism must be true. Physicists are willing to contemplate indeterministic theories: arguably, quantum mechanics is indeterministic. But they do think that the particular sort of indeterminism revealed by the hole argument is unacceptable. Specifically, their point is that

> [I]f a metaphysics, which forces all our theories to be deterministic, is unacceptable, then equally a metaphysics, which automatically decides in favour of indeterminism, is also unacceptable. Determinism may fail, but if it fails it should fail for a reason of physics, not because of a commitment to substantival properties which can be eradicated without affecting the empirical consequences of the theory. ((Earman & Norton, 1987), p.524)

[2] Even more radically, we can hold fixed the entire manifold save some arbitrarily small 'hole', and consider deformations of the manifold in the interior of the hole. Via this consideration, a precisely analogous argument leads to the conclusion that, according to the substantivalist, fixing the values of all fields on all of spacetime except for an arbitrarily small hole does not suffice to fix the value of any field at any point within the hole. It is this version of the argument, of course, that earned it the name 'hole argument'.

## 6 Standard substantivalist responses to the hole argument

The 'hole argument' of the preceding section can be summed up as follows:

**P1.** There are models of general relativity that differ on which field values are instantiated at which manifold points within a 'hole', but not elsewhere.

Let $M, M'$ be such a pair of diffeomorphically-related models.

**P2.** Substantivalism is committed to the claim that $M$, $M'$ represent physically distinct situations.

**P3.** Substantivalism is committed to the claim that $M, M'$ each represent a physically possible situation.

**P4** If $M, M'$ represent distinct physically possible situations, then indeterminism holds.

Therefore,

**C.** Substantivalism is committed to indeterminism.

This argument is clearly valid. Of its premises, (P1) and (P4) are unassailable: P1 follows immediately from the fact that general relativity is diffeomorphism invariant, and P4 follows immediately from the definition of indeterminism. The substantivalist has three options: deny P2, deny P3, or bite the bullet. Each of these has been proposed by defenders of substantivalism.

The bullet-biter is the **haecceitistic indeterminist** (e.g. (Brighouse, 1994)). Advocates of this response distinguish *haecceitistic* from *non-haecceitistic* indeterminism. A theory is haecceitistically indeterministic iff it has models that agree on field values at all spacetime points up to some time, and disagree after that time by permutation of spacetime points; it is *non-haecceitistically indeterministic* iff it has models that agree on field values at all spacetime points up to some time, and disagree after that time by more than mere permutation of spacetime points. Then, the idea is that the hole argument establishes only haecceitistic indeterminism, while it is only non-haecceitistic indeterminism that should not follow from metaphysics alone.

The P3-denier is the **sophisticated substantivalist** (e.g. (Pooley, 2006)). Advocates of this response deny that spacetime points *have* haecceities. It is supposed to follow from this that two diffeomorphically-related solutions represent the same physical situation; that is, that **Leibniz equivalence** holds.

The P2-denier is the **metric essentialist** (e.g. (Maudlin, 1989)). Metrical essentialism is the view that spacetime points have their metric properties essentially. This entails that if some particular model represents a metaphysically possible situation, a model related to the first by a non-trivial diffeomorphism in general will not (since it will, relative to the first model, represent one and the same spacetime point as having different metrical properties, in violation of that point's essence).

There is plenty to be said, and plenty has been said, about the clarification, and the absolute and relative merits and demerits, of these positions, and of how relationist alternatives might fare under similarly close scrutiny. We will not enter into this discussion, since our present interest is rather whether the 'traditional debate' that we have just reviewed is either missing some radically different but plausible option, or in any other way based on a misconception.

## 7 Voices of dissent

The clearest statement of the view that the substantivalist/relationist debate is inappropriate in these general-relativistic times is probably Robert Rynasiewicz's:

> In the course of its development, physical theory simply lost touch with the categories necessary for the original formulation of the [substantivalist-relationist] problem. One might, of course, seek judicious criteria for projecting old categories onto new terrain. [But] one should be skeptical that there is any such natural or preferred projection in this case. ... [T]he current [substantivalist-relationist] controversy reduces to verbal disputes occasioned by arbitrary preference for one manner of projection over another. (Rynasiewicz, 1996a), p.279

Several authors think that the more enlightened position is some sort of *structuralism*. As Mauro Dorato summarizes the trends of the recent literature:

> Difficulties to adjust the substantivalism/relationism dichotomy to the framework of the General Theory of Relativity ... have

> pushed philosophers of space and time to find alternative formulations of the debate. Among these, various forms of structural spacetime realism — more or less explicitly formulated — have been proposed either as a third stance between the two age-old positions ([(Stachel, 2002), (French & Rickles, 2006), (Esfeld & Lam, 2006)]), or as an effective way to overcome or dissolve the substantivalism/relationism debate ([(Stein, 1967), (DiSalle, 1995), (Dorato, s. d.), (Dorato & Pauri, 2006), (Slowik, 2005)]). (Dorato, 2007, p.1)

Dorato himself favours a structuralism of the second sort:

> If we agree in stipulating that "spacetime exists iff the physical world exhibits the corresponding spatiotemporal structure", I would like to press the point that the empirical success of our spacetime models do raise an important ontological question ("does spacetime exist?"), while the particular manner of existence of spacetime, namely whether it is substance-like or relation-like, after the establishment of [general relativity] has become a less central, *metaphysical, possibly merely verbal* question. (Dorato, 2007, p.19; emphasis in original)

The remainder of this paper is an attempt to disentangle, and assess the merits of, several lines of thought in the 'structuralist' literature.

## 8 What spacetime structuralism might be

**Structuralism v1: avoidance of outdated metaphysical categories.** Much of the (seventeenth-century and recent) substantivalist-relationist discussion has been carried out in terms of concepts that my above formulations of the dispute (in section 3) did not mention. The reason I did not mention them is that I myself find it difficult to make sense of them, or to state any clear question or disagreement in their terms. It is worth drawing attention to this, because *some* of the structuralist literature is most naturally read as making precisely the point that the concepts in question are not determinate, important and/or sharp enough to be useful.

To elaborate: In setting up the substantivalist-relationist dispute in section 3, I phrased things exclusively in terms of whether or not spacetime points are *objects*. This is a rather minimal version of 'the' substantivalist-relationist question. Substantivalists have traditionally been concerned with such issues as (i) whether spacetime is a *substance* (where this 'substancehood' is supposed to be a thicker concept than that of mere objecthood), (ii) whether spacetime can be regarded as the 'container' in which material objects are 'contained', and (iii) whether spacetime is to be identified with the manifold alone or the manifold plus gravitational (i.e. metric) field. These, I think, are non-issues, for such various reasons as (i′) that the notion of substancehood-over-and-above-objecthood is hopelessly obscure, (ii′) that they can be stated only in metaphorical terms and hence are not well-posed questions, and (iii′) that they are merely verbal questions, with (iii′(a)) no determinate answers (existing usage failing to have resolved semantic indecision) and also (iii′(b)) no theoretical importance.

This deflation of *parts of* the substantivalist-relationist debate provides a natural reading of Rynasiewicz's comment (quoted above), and also of Dorato's remark that

> [T]he duality of the metric field and the difficulties of defending a "container/contained", or a "spacetime/physical field" distinction in classical [general relativity] speak definitely in favour of a dissolution of the substantivalism/relationism debate. (Dorato, 2007), p.3

*This* point is well taken. However, it should be clear that it will not dissolve the *whole* of the substantivalist-relationist debate. There still remains the question, utterly unimpugned by the rejection of such things as a thicker concept of substance and the 'metaphysical' status of the metric field as 'field on' versus 'property of' versus 'part of' spacetime, of whether spacetime points are among the objects in which the success of our best spacetime theories should lead us to believe.

**Structuralism v2: deflation of the object-property distinction.** A more radical rejection of traditional metaphysics, and one that *would* dismiss even this residue of the substantivalist-relationist issue as a pseudo-question, holds that there is no matter of fact as to which features of the world are objects and which are properties (incl. relations). The view might be that while language (both natural and formal) compels us to place some features of the world in subject (or object) position and others in predicate (or property) position, the division does not correspond to any fundamental cleavage of the furniture of the world into corresponding categories. ('Socrates is wise,' most will say; but those who prefer to say that 'Wisdom is Socrified' speak no less truly; cf. (Ramsey, 1925).)

Something like this view is apparently suggested by Dorato's remark that

> A one-sentence way of putting the main point of this paper would be the following: spacetime exists as exemplified structure, while the question whether it exists as substance or relation is not well-posed. (Dorato, 2007, pp.19-20)

This is obviously a sweeping metaphysical claim. I have some sympathy with it, but one does worry (a) that the appeals to 'structure-exemplification' and 'world-features that can be represented this or that way in object-property terms but do not themselves dictate any preferred representation' are mere obscurantism, and that the view gains any plausibility it may appear to have only thanks to unclarity; (b) that the decision of whether to treat something as object or property *will* turn out to have some theoretical importance (no matter that it is currently unclear what sort of importance this might be) that the examples of Socrates and wisdom fail to exemplify, and hence that a deflationism that denies this possibility deflates too much. One cannot help being reminded of the fate of logical positivism: there was something very plausible-sounding about its basic ideas, but the devil was in the details, and by stating it clearly we saw how it failed.

Further attempt to develop and/or assess the merits of this intriguing suggestion lie outside the scope of the present paper. But in any case, version 2 is apparently not what most structuralists take themselves to be saying, since (a) it has nothing in particular to do with general relativity, (b) no other author in this tradition comes anywhere near as the above Dorato quote to *saying* that this is what his point is supposed to be, and, as we will see, (c) many of the authors in question crucially *rely* on the fundamentality of the object/property/relation distinction.

**Structuralism v3: Spacetime points as abstract roles in isomorphism classes.** The idea of our third brand of structuralism about spacetime points treats such points as mathematical structuralists treat numbers. That is, it holds that spacetime points are to be regarded, not as objects, but as 'structural roles in isomorphism classes of models of certain theories' (Mundy, 1992, p.523).

As Rynasiewicz (1996b) points out, the analogy with mathematics is misleading in one important respect. To say that numbers are abstract roles is not to deny that they exist or that they fall into the broad ontological category that any philosopher of mathematics would take them to fall into, viz. that of abstracta. Physical spacetime points, in contrast, are taken by the spacetime realist to be *concrete* objects. Abstract-role structuralism about spacetime points therefore amounts to *antirealism* about spacetime points, in the terms of the spacetime debate.

Rynasiewicz emphasizes this point because, according to him, Mundy himself proposed his form of 'structuralism' as a way of showing, against frequent suggestions to the contrary, that Leibniz Equivalence is consistent with *realism* about spacetime points. Mundy's structuralism therefore does not get him where he thought it would get him. But putting that aside, we can ask: does it get us anywhere useful or interesting?

It seems likely this sort of structuralism will collapse into relationism. For the relationist, too, holds that *physical* spacetime points do not exist, and that what is real corresponds to the relations that are shared by all elements of an isomorphism class of models of a spacetime theory. Mundy's structuralism, in that case, is neither a third way nor a debate-dissolver.

**Structuralism v4: 'Moderate structural realism' about spacetime.** In much of the substantivalist-relationist discussion, there has been a curious refusal to accept the possibility that substantivalists can consistently deny that manifold points have haecceities. Our next brand of structuralism embeds the claim that this possibility is actual into an independent motivating story, connecting the repudiation of haecceities to wider issues in the philosophy of science.

The refusal to countenance 'sophisticated substantivalism' is sometimes uncritical, sometimes argued for. For an example of uncritical insistence, witness, for example:

> Whatever reformulation a substantivalist may adopt, they must all agree concerning an *acid test of substantivalism*, drawn from Leibniz. If everything in the world were reflected East to West (or better, translated 3 feet East), would we have a different world? The substantivalist must answer yes *since all the bodies of the world are now in different spatial locations*, even though the relations between them are unchanged. (Earman & Norton, 1987, p.521; emphasis added)

In this passage, Earman and Norton appear to be ruling out sophisticated substantivalism simply because the possibility has failed to occur to them. But the idea that sophisticated substantivalism is somehow incoherent, and that the denial of haecceities leads inexorably to antirealism about spacetime points, survived its explicit formulation; for further argument, see, for example, (Rynasiewicz, 1994, pp. 416-7); (Belot & Earman, 2001, pp. 248-9).

Meanwhile: an independently motivated 'structural realist' movement in the philosophy of science has been seeking a 'third way' between traditional scientific realism (implausible, perhaps, in the face of the pessimistic meta-induction) and instrumentalism (which seems overly cautious), by claiming that the success of our best scientific theories should lead us to believe, not that they are true simpliciter or 'approximately true', but that the claims they make about the 'structure' of physical reality are true. The thesis comes in epistemic and ontic variants: the epistemic variant claims that the 'structure' of physical reality is *all we can have justified belief in*, while the ontic variant claims that the 'structure' is *all there is*. The ontic claim ('structure is all there is') is then sometimes cashed out as the claim that *there are no intrinsic properties*. (This is the position that Esfeld and Lam (2006) dub 'moderate structural realism'.) And haecceities are surely intrinsic properties if anything is. Applying this general line of thought to general relativity and the hole argument in particular, the intended upshot is clear: in (moderate ontic) structural realism we have an independently motivated claim that spacetime points do not have haecceities.

This is all very well, but the contribution of structuralism in this case should not be overestimated: the original sophisticated substantivalist will welcome any sound independent motivation for his key thesis, but independent motivation is not a new thesis. Structuralism seems, once again, to be a footnote to one of the existing parties to the traditional debate, and neither a third way nor a debate-dissolver.

**Structuralism v5: 'Radical structural realism' about spacetime.** During the broader discussion of structural realism in the philosophy of science, a 'radical structural realism' (to be contrasted with Esfeld and Lam's 'moderate structural realism') also emerges.

The radical structural realist holds that *there are no objects*. He cashes out 'structure' as 'a system of relations', and his ontology is supposed to be one of relations without relata. (This position has been advocated by, e.g. Steven French and James Ladyman (2003).)

*If* coherent, radical structural realism about spacetime (RSRS) would indeed be a 'third way': a genuine alternative to both substantivalism (in any of the variants listed above) and relationism. RSRS would be the thesis that spacetime is a primitive system of relations. It would not be any sort of substantivalism, since it would deny that any objects were spacetime points (and, indeed, that spacetime points exist at all). But it would not be relationism as defined above either, since relationism was supposed to be the thesis that spacetime is a set of relations *between material bodies or field-parts*, whereas RSRS denies that the spatiotemporal relations have relata at all.

Radical structural realism, about spacetime points or about anything else, faces a widespread complaint that its notion of 'relations without relata' is incoherent. This is not the place to attempt to adjudicate that dispute; we merely record that RSRS's promise to provide a 'third way' between substantivalism and relationism is hostage to its promissory note that this complaint can be satisfactorily answered.

**Structuralism v6: Individuation of spacetime points via metrical properties.** Our sixth version of structuralism centres on the notion of criteria of individuation. Spacetime points, our last type of structuralist says, are individuated via the metric, and only via the metric. Therefore (the argument runs), Leibniz equivalence holds, and so the hole argument is defused. For example:

> A wide range of philosophers of physics and physicists agree on the fact that [the radical indeterminism suggested by the hole argument] is a consequence of the non-physical primary individuation of space-time points independently of the metric. This unpalatable feature can hence be avoided by claiming that *there is no physical individuation within [general relativity] of space-time points independently of the metric.* Indeed, this can be seen as the moral of the fundamental [general relativity] principle of active general covariance. (Esfeld & Lam, 2006), pp. 9-10

> The following possibilities have emerged concerning the individuation of the objects in a set with structure . . . : 1) They are individuated independently of their position in [a] relational structure . . . . 2) They are only individuated . . . by their position in the relational structure.

> The hole argument is invalid (or can be avoided by proper formulation of the theory in . . . [cases] in which the relational structure confers all the relevant individuality to the things between relations. (Stachel, 2002), pp.247-8

> *[I]f* we think of the points of [the hole] as *intrinsically individuated physical events*, where "intrinsic" means that their identity

> is independent of the metric — a claim that is associated with *manifold substantivalism* — then [diffeomorphically-related solutions] must be regarded as *physically distinct* solutions of the Einstein equation ... (Dorato & Pauri, 2006), p.9; emphasis in original;

The theme of criteria of individuation is taken up in criticism of spacetime structuralism by Wuthrich (2009), who argues that structuralists are committed to the impossibility of highly symmetric spacetimes (on the grounds that their criteria of individuation commit them to regarding 'all' the points in such spacetimes as identical, i.e. to regarding such spacetimes as containing only a single point). Muller (s. d.) replies that even in such highly symmetric spacetimes distinct points are *weakly* discernible, and that this suffices.

Clarification is required: there is more than one thing that such talk of 'individuation' could mean. But herein lies a dilemma; on resolving it, our sixth version of spacetime structuralism will either collapse into sophisticated substantivalism, or fail to entail Leibniz Equivalence.

To elaborate: there are two quite distinct claims that might be intended by the phrase 'spacetime points are individuated only via the metric'. The first claim is made in the context of a view that intra-world identity and distinctness cannot be primitive — that, if two objects are distinct, this cannot be a brute fact, but must be grounded in some property/ies or relation(s).[3] The suggestion is then that metrical properties/relations provide the ground for the fact that a given spacetime contains infinitely many points, rather than (say) just one.

On the second claim, the issue has nothing directly to do with *intra*-world distinctness, but rather concerns the semantics for 'might' counterfactuals (such as 'it might have been that for every point $p$ in region $R$, $p$ had the metrical properties that are actually possessed by $f^{-1}(p)$').

The precise details of this story vary depending on whether one adopts a Kripke-style or a counterpart-theoretic semantics for such counterfactuals, but the broad outline is the same either way. All parties agree that there is

[3] Thus the view is that some form of intra-world principle of identity of indiscernibles *must* be true, on pain of incoherence. Why one might hold this view is not clear. For some authors, the motivation seems to be epistemological or methodological: that we could have no way of knowing that, or grounds for postulating that, we were dealing with two objects rather than one unless the distinctness had some such grounding. This motivation seems to me to rely on an overly naive epistemology (for some discussion of this point, in the context of a (mathematical) example of the coherent failure of all intra-world PIIs, see (Leitgeb & Ladyman, 2008)). More often no motivation is discussed or hinted at.

a relation $R$ such that: p might have had metrical properties $F$ iff there is a world $w'$ and a point $p'$ in $w'$ such that $p'$ bears relation $R$ to $p$ and $p'$ has $F$. The Kripke-semanticist then adds that $R$ is simply the identity relation; the counterpart theorist disagrees, holding instead that it is a counterpart relation. The individuation claim is then a claim that metrical properties suffice to determine (respectively) transworld identity or counterparthood. (In the former case, it is thus a version of the *trans*-world principle of identity of indiscernibles.)

Which of these claims — first or second— is intended? Wuthrich and Muller clearly have in mind the first claim; more often it is not clear. But in any case, any attempt to answer the question on behalf of the structuralists faces a dilemma. On the first reading, it is unclear how the individuation claim is supposed to lead to Leibniz equivalence[4]; so this version does not succeed in defusing the hole argument. On the second reading, Leibniz Equivalence plausibly does follow, but the structuralism we end up with is just an explication of sophisticated substantivalism. In neither case do we have a new *and* successful defusing of the hole argument.

## 9 Conclusions

Considerations that could plausibly be labelled 'structuralist' (but many of which are incompatible with one another) interact with the traditional substantivalist-relationist dispute in at least four ways.

'Structuralism' could be taken to be the thesis ('version 1') that the only well-posed core of the substantivalist-relationist question is that of whether or not spacetime points are objects; the point here is to rule out, as merely verbal, (e.g.) questions about whether the metric field is 'part of spacetime', a 'property of spacetime' or a 'field on spacetime', and questions requiring the geometry/matter or container/contained distinctions to be taken too seriously. *This* claim seems to me to be correct. However, it does not dissolve the remaining substantivalism-relationalism debate as set out in this paper.

Alternatively, structuralism could be the thesis ('version 2') that even that core is not well-posed, because there is no privileged way of carving up the world-structure into objects and properties. This is a far more sweeping

[4]A trans-world PII does entail an intra-world PII, provided that the PII is really a principle of *identity* of indiscernibles, not mere counterparthood (so that we are guaranteed to have transitivity). But, crucially for our purposes, the intra-world PII does not entail any trans-world PII.

claim, and *could* be correct; it is interesting and warrants further investigation, but in any case goes well beyond the bounds of what general relativity and/or contemplation of diffeomorphism invariant theories in general has taught us.

We have canvassed also four further possible 'structuralisms'. Versions 4 (moderate structural realism) and 6(i) ('metrical individuation' in the sense of counterpart theory) turned out to be mere elaborations of sophisticated substantivalism, and so added nothing radically new to the traditional debate. Version 3 (mathematical structuralism about spacetime points) seems likely to collapse into relationism. Version 5 (radical structural realism about spacetime) involves anti-realism about spacetime, but it is *prima facie* obscure what the advocate of this view *does* take to be physically real. I have questioned the motivation for Version 6(ii) ('metrical individuation' in the sense of intra-world PII), but in any case this thesis has no apparent relevance to the hole argument.

Version 2 thus seems the best suited to achieving the aim of debate-dissolver, but has daunting work to do to earn the status of positive thesis. The point of the present paper, in any case, is that the above six theses are *distinct* and, moreover, do not appear to be closely related; to conflate them can only lead to confusion. It further seems to me (although I have not argued this here) that they *are* often conflated; if so, clarity would perhaps be served by a ten-year moratorium on use of the word 'structuralism'.

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